Hey everyone! My name is Savanah Smith, and I am the Master Tutor for Calculus III this semester. I hope these resources can become a great studying and review tool for everyone who encounters them. I will also be having weekly Group Tutoring sessions where we will go over the topics and practice problems covered in these resources, so feel free to check that out as described below! Please don’t hesitate to reach out to me with any questions or comments or when in doubt go to the tutoring website!

Group Tutoring: Mondays from 5:15 – 6:15 pm
In Sid Rich Basement, Room 75
Reserve a spot at baylor.edu/tutoring

This resource will be covering the highlights learned throughout the semester to help with cumulative final exams!

**Key Words:** Final Review

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**LIST OF IMPORTANT TOPICS**
*(with referenced resources)*

**Vectors** *(Week 3)*
- Dot and Cross Product

**Different Coordinate Systems** *(Week 3)*
- Rectangular / Polar / Cylindrical / Spherical

**Vector Valued Functions** *(Week 4)*

**Arc Length** *(Week 4)*

**Partial Derivatives** *(Week 5)*

**Tangent Planes** *(Week 5)*

**Gradients** *(Week 6)*

**Directional Derivatives** *(Week 6)*

**Optimization** *(Week 6)*

**Double Integrals** *(Week 7)*

**Triple Integrals** *(Week 8)*

**Vector Fields** *(Week 9)*
- Curl and Divergence
- Conservative

**Line Integrals** *(Week 10)*

**Surface Integrals** *(Week 11)*

**Greens Theorem** *(Week 12)*

**Stokes Theorem** *(Week 13)*

**Divergence Theorem** *(Week 15)*
Important Equations and Operations

Vectors
Magnitude: \( \| \overrightarrow{PQ} \| = \sqrt{x^2 + y^2} \) where \( \overrightarrow{PQ} = (x, y) \),

Dot Product: between two vectors, produces a scalar
\[
\mathbf{a} = \langle a_1, a_2, a_3 \rangle \quad \mathbf{b} = \langle b_1, b_2, b_3 \rangle
\]
\[
\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3
\]

Cross Product: between two vectors, produces a vector \( \langle i, j, k \rangle \)
\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
i & j & k \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
\end{vmatrix} = (a_2 b_3 - a_3 b_2)i - (a_1 b_3 - a_3 b_1)j + (a_1 b_2 - a_2 b_1)k
\]

Different Coordinate Systems

Cylindrical \( (r, \theta, z) \)

Rectangular \( (x, y, z) \)

Spherical \( (\rho, \theta, \phi) \)

Arc Length
\[
s = \int_a^b \sqrt{x'(t)^2 + y'(t)^2} \, dt
\]

Gradients & Directional Derivatives
\[
\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle \quad D_{\mathbf{u}} f (P) = \nabla f_p \cdot \mathbf{u}
\]
Vector Fields
Divergence: how much the flow is expanding ($\text{div}(F) < 0$) or compressing ($\text{div}(F) > 0$)

$$\text{div}(F) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Curl: how the flow rotates

$$\text{curl}(F) = \left( \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right) \mathbf{i} - \left( \frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z} \right) \mathbf{j} + \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \mathbf{k}$$

Line Integrals
Scalar: find total mass, total charge density, etc.

$$\int_C f(x, y, z) \, ds = \int_a^b f(\mathbf{r}(t)) \|\mathbf{r}'(t)\| \, dt$$

Vector: find work

$$\int_C \mathbf{F} \, d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt$$

Surface integrals: $\mathbf{N}$ is the normal vector

$$\int_S f(x, y, z) \, dS = \int_D f(G(u, v)) \|\mathbf{N}(u, v)\| \, du \, dv$$

Important Concepts

Partial Derivatives
Finding the rate of change with respect to each variable separately

Notation: $f_{xy} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right)$

The most important key to determining the partial derivatives is to know which variable you are taking the derivative of and treating the other variable(s) as constants.

Optimization
Finding the extreme values of a function

Step 1: Find critical points using partial derivatives

$$f_x(a, b) = 0 \text{ and } f_y(a, b) = 0 \text{ or do not exist}$$

Step 2: use second derivative test to determine the type of critical points
\[ D = f_{xx}(a, b) f_{yy}(a, b) - f_{xy}^2(a, b) \]

(i) If \( D > 0 \) and \( f_{xx}(a, b) > 0 \), then \( f(a, b) \) is a local minimum.
(ii) If \( D > 0 \) and \( f_{xx}(a, b) < 0 \), then \( f(a, b) \) is a local maximum.
(iii) If \( D < 0 \), then \( f \) has a saddle point at \((a, b)\).
(iv) If \( D = 0 \), the test is inconclusive.

**Double / Triple Integrals**
Subsequent integration of one variable at a time
Used to find volume of a 3D shape

\[
\iiint_B f(x, y, z) \, dV = \int_{x=a}^{b} \int_{y=c}^{d} \int_{z=p}^{q} f(x, y, z) \, dz \, dy \, dx
\]

**Green's Theorem**
For non-conservative 2D surfaces in simple closed curves

\[
\oint_{\partial D} F_1 \, dx + F_2 \, dy = \iint_D \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \, dA
\]

**Stokes' Theorem**
For non-conservative 3D surfaces that have single closed curves

\[
\oint_{\partial S} F \cdot dr = \iint_S \text{curl}(F) \cdot dS
\]

**Divergence Theorem**
Solves for flow rate or flux of a closed 3D surface

\[
\iiint_V \text{div}(\mathbf{F}) \, dV = \iiint_S \mathbf{F} \cdot dS
\]
CHECK YOUR LEARNING

1. Find the dot and cross product of the vectors $S = \langle 1,3,5 \rangle$ and $T = \langle 4,3,6 \rangle$
2. Find the partial derivative in terms of $x$ and then $y$ of $f(x,y) = e^{-x} + \sin(x + 2y)$
3. Evaluate $\iiint_W z \, dV$ where $W$ is the region between the planes $z = x + y$ and $z = 3x + 5y$ over the rectangle $D = [0,3] \times [0,2]$.
4. Find the divergence and curl of $F = \langle xy, e^x, y + z \rangle$.

THINGS YOU MAY STRUGGLE WITH

1. There is a lot to remember, especially for a comprehensive final exam but hopefully this will help provide a list of the general equations and concepts to know. With these, practice looking at problems, figuring out what you need to find, and detailing the path to get there. Being able to connect what a question asks to what you need to do will help you immensely.
2. For any problem, take extra time and care to set up the problem correctly. Make sure you have the right variables, the right bounds, the right operations, etc. Even if you aren’t sure exactly how to solve a problem if you can at least set up the calculus parts of it, you will get more points compared to setting it up incorrectly.

That’s all for this semester! I hope this is a helpful review and GOOD LUCK ON YOUR FINALS! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. $433\mathbf{i} + 14\mathbf{j} - 9\mathbf{k}$
2. $f_x = -e^{-x} + \cos(x + 2y), f_y = 2\cos(x + 2y)$
3. 294
4. $\text{div} = y + 1, \text{curl} = \mathbf{i} + (e^x - x)\mathbf{k}$