Calculus 1, Week 14

Hey Calculus tutors and students! This resource covers the fourteenth week of class. Because less material is covered this week, I will be including some extra review questions on last week's topics. In addition, I will go over how to take integrals that become inverse trig functions – a topic introduced in 5.8.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

Keywords: Fundamental Theorem of Calculus (2nd part), Distance, Displacement, U-substitution, Inverse Trig Integration.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

Chapter 5.5 Review

- Example Problems
    - What is the derivate of $G(x) = \int_2^{\sqrt{x}} cos(t)\, dt$
        - $G(x) = A(\sqrt{x}), where\ A(x) = \int_2^{x} cos(t)dt$, so $G'(x) = A'(\sqrt{x}) * \left(\frac{1}{2\sqrt{x}}\right)$. Since $A(x) = \int_2^{x} cos(t)dt$, $A'(\sqrt{x}) = cos(\sqrt{x})$. Thus, $G'(x) = cos(\sqrt{x}) * \left(\frac{1}{2\sqrt{x}}\right)$.
    - What is the derivate of $G(x) = \int_7^{5x^4} \sqrt{t}\, dt$
        - $G(x) = A(5x^4), where\ A(x) = \int_7^{x} \sqrt{t}\, dt$, so $G'(x) = A'(5x^4) * (20x^3)$. Since $A(x) = \int_7^{x} \sqrt{t}\, dt$,

$A'(5x^4) = \frac{2}{3}(5x^4)^{3/2}$. Thus, $G'(x) = \frac{2}{3}(5x^4)^{3/2} * (20x^3)$.

Chapter 5.6 Review

- Example Problems
  - What was the total displacement of a person moving at velocity $= t^3 - 9t$ from t=1 to t=7? What was the distance?
    - Displacement $= \int_1^7 (t^3 - 9t)dt = 384$
    - Distance $= \int_1^7 |(t^3 - 9t)|dt = -\int_1^3 (t^3 - 9t)dt + \int_3^7 (t^3 - 9t)dt = 416$
      - The function $t^3 - 9t$ is negative from [1,3], so I had to split the integral into two pieces and manually ensure that both pieces would be positive.
    - Follow up question: in the above example we got a larger distance than displacement. Does this make sense? Is there any time that displacement would be greater than distance?
      - A: It does make sense, and distance is always greater than or equal to displacement.
  - What was the total displacement of a person moving at velocity $= t^2 - 8t$ from t=0 to t=6? What was the distance?
    - Displacement $= \int_1^{10} (t^2 - 8t)dt = -63$
      - Does it make sense that we would have a negative displacement?
        - Yes – displacement can be negative. It is distance that is always positive.
    - Distance $= \int_1^{10} |(t^2 - 8t)|dt = -\int_1^8 (t^2 - 8t)dt + \int_8^{10} (t^2 - 8t)dt =$ 583

Chapter 5.7 Review

- Example Problems
  - $\int_2^9 x^2 \cos x^3 \, dx$
    - *Set* $u(x) = x^3$ *and* $du = 3x^2dx$.
      As such we can rewrite the integral as $\int_{u(2)}^{u(9)} \cos u * \frac{du}{3}$.
      $\int_{u(2)}^{u(9)} \cos u * \frac{du}{3} = \frac{1}{3}\sin(u(9)) - (\frac{1}{3})\sin(u(2)) = \frac{1}{3}\sin(729) + \frac{1}{3}\cos(8)$
  - $\int_1^7 x \, e^{x^2-7} \, dx$
    - *Set* $u(x) = x^2 - 7$ *and* $du = 2xdx$.
      As such we can rewrite the integral as $\int_{u(1)}^{u(7)} e^u * \frac{du}{2}$
      $\int_{u(1)}^{u(7)} e^u * \frac{du}{2} = \frac{1}{2}e^{u(7)} - \left(\frac{1}{2}\right)e^{u(1)} = \frac{1}{2}e^{42} + \frac{1}{2}e^6$

- $\int_{-4}^{2} e^x\, sec(e^x)\, tan(e^x) dx$
  - $Set\ u(x) = e^x$ and $du = e^x dx. As\ such$ $we\ can\ rewrite\ the\ integral\ as\ \int_{u(-4)}^{u(2)} sec(u)tan(u) * du$ $\int_{u(-4)}^{u(2)} sec(u)tan(u) * du = sec(u(2)) - sec(u(-4)) = sec(e^2) - sec(e^{-4})$
- $\int_{0}^{10} (x^2+1)(x^3+3x)^3 dx$
  - $Set\ u(x) = x^3 + 3x$ and $du = (3x^3+3)dx$.
    $As\ such\ we\ can\ rewrite\ the\ integral\ as\ \int_{u(0)}^{u(10)} u^3 * \frac{du}{3}$
    $\int_{u(0)}^{u(10)} u^3 * \frac{du}{3} = \frac{1}{3*4} u(10)^4 - \frac{1}{3*4} u(0)^4 = \frac{1}{12}(1030)^4 - \frac{1}{12}(0)^4$

Chapter 5.8

- In chapter 5.8 the students learn how to integrate a few more functions. Specifically, they learn
  - $\int \frac{dx}{x} = lnx + c$
  - $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$
  - $\int \frac{dx}{x^2+1} = \tan^{-1} x + c$
  - $\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + c$
  - As it turns out, due to the equation of inverse cos, cot, and csc, the integrals that would produce these functions are identical to the three inverse trig functions above, except with a negative sign in front of the integral. As such, you don't need memorize them independently as long as you can remember how they are similar to the definitions above.
- New Video Resource
  - https://www.youtube.com/watch?v=CFmTDn7BQmE
- Example Problems
  - Solve $\int \frac{xdx}{x^4+9}$
    - $\int \frac{xdx}{x^4+9} = \int \frac{xdx}{\left(\frac{x^4}{9}\right)+1}$
      set u= $\frac{1}{3}x^2$ and $du = \frac{2}{3}xdx$
      $\frac{3}{2}\int \frac{du}{u^2+1} = \frac{3}{2}\tan^{-1} u + c = \frac{3}{2}\tan^{-1}(\frac{1}{3}x^2) + c$
  - Solve $\int \frac{x^2dx}{\sqrt{1-5(x^6)}}$
    - $\int \frac{x^2dx}{\sqrt{1-5(x^6)}} = \int \frac{x^2dx}{\sqrt{1-(\sqrt{5}x^3)^2}}$
      set u= $\sqrt{5}x^3$ and $du = 3\sqrt{5}x^2dx$

$$\frac{1}{3\sqrt{5}}\int\frac{du}{\sqrt{1-u^2}}=\frac{1}{3\sqrt{5}}\sin^{-1}u+c=\frac{1}{3\sqrt{5}}\sin^{-1}\sqrt{5}x^3+c$$

- Solve $\int\frac{\sqrt{x}dx}{x^{3/2}}$
  - $\int\frac{\sqrt{x}dx}{x^{3/2}}$

    set u= $x^{3/2}$ *and* $du=(\frac{3}{2})x^{1/2}dx$

    $\frac{2}{3}\int\frac{du}{u}=\frac{2}{3}ln(u)+c=\frac{2}{3}ln(x^{3/2})+c$

- Solve $\int\frac{x^3dx}{|\sqrt{7}x^4|\sqrt{7x^8-1}}$
  - $\int\frac{x^3dx}{|\sqrt{7}x^4|\sqrt{(\sqrt{7}x^4)^2-1}}$

    set u= $\sqrt{7}x^4$ *and* $du=4\sqrt{7}x^3dx$

    $$\frac{1}{4\sqrt{7}}\int\frac{du}{|u|\sqrt{(u)^2-1}}=\frac{1}{4\sqrt{7}}\sec^{-1}u+c=\frac{1}{4\sqrt{7}}\sec^{-1}(\sqrt{7}x^4)+c$$

- Solve $\int\frac{x^3dx}{x^8+25}$
  - $\int\frac{x^3dx}{x^8+25}=\int\frac{x^3dx}{\left(\frac{x^8}{25}\right)+1}$

    set u= $\frac{1}{5}x^4$ *and* $du=\frac{4}{5}x^3dx$

    $\frac{5}{4}\int\frac{du}{u^2+1}=\frac{5}{4}\tan^{-1}u+c=\frac{5}{4}\tan^{-1}(\frac{1}{5}x^4)+c$

- Solve $\int\frac{xdx}{\sqrt{1-7(x^4)}}$
  - $\int\frac{xdx}{\sqrt{1-7(x^4)}}=\int\frac{xdx}{\sqrt{1-(\sqrt{7}x^2)^2}}$

    set u= $\sqrt{7}x^2$ *and* $du=2\sqrt{7}xdx$

    $\frac{1}{2\sqrt{7}}\int\frac{du}{\sqrt{1-u^2}}=\frac{1}{2\sqrt{7}}\sin^{-1}u+c=\frac{1}{2\sqrt{7}}\sin^{-1}\sqrt{7}x^2+c$

- Solve $\int\frac{x^3dx}{|\sqrt{7}x^4|\sqrt{16(x^2)-1}}$
  - $\int\frac{x^3dx}{|4x|\sqrt{(4x)^2-1}}$

    set u= $4x$ *and* $du=4dx$

    $$\frac{1}{4}\int\frac{du}{|u|\sqrt{(u)^2-1}}=\frac{1}{4}\sec^{-1}u+c=\frac{1}{4}\sec^{-1}(4x)+c$$

Things Students Tend to Struggle With

- **Integrals of Inverse Trig Functions**
  - Unfortunately, the integrals that become inverse trig functions are just one of the things in Calculus 1 that you end up having to memorize. One nice thing about them is that they look very similar to one another – for instance, the derivatives of arcsin and arccos are the same except for a negative sign. Similarly, they all appear as fractions, and most of them have radicals in the denominator. So if you see a problem that requires you to integrate a fraction, one of the first things you should do is consider if your integral looks like it might become an inverse trig function.