Calculus 1, Week 13

Hey Calculus tutors and students! This resource covers the thirteenth week of class. This week I will be covering the 2nd part of the Fundamental Theorem of Calculus. In addition, I will go over how to calculate distance and displacement with integrals, as well as how to use U-Substitution to calculate integrals.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

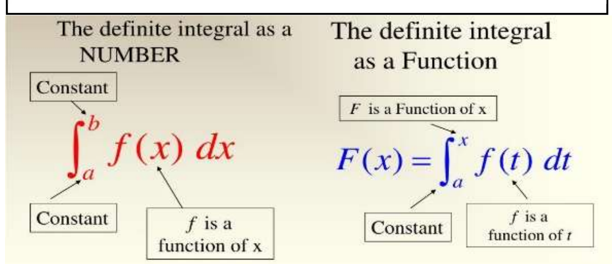
- Alex Williamson

Keywords: Fundamental Theorem of Calculus (2nd part), Distance, Displacement, U-substitution.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Another way to represent the 2nd part of the Fundamental Theorem of Calculus



Source: https://www.slideserve.com/rue/average-value-of-a-function-and-the-second-fundamental-

Concepts

Chapter 5.5

- In chapter 5.5 the students are learning the 2nd part of the fundamental theorem of calculus. The second part of the fundamental theorem of calculus states that $\frac{d}{dx}\int_a^x f(t)dt = f(x)$. Also, as an implication, if $G(x) = A(g(x)) = \int_a^{g(x)} f(t)dt,\ then\ G'(x) = A'(g(x)) * (g'(x)) = g'(x)f(x)$
- Video Resource
    - https://www.youtube.com/watch?v=q_PNdqlvfPw

- Example Problems
  - What is the derivate of $G(x) = \int_{-2}^{x^3} e^t \, dt$
    - $G(x) = A(x^3), where\ A(x) = \int_{-2}^{x} e^t dt$, so $G'(x) = A'(x^3) * (3x^2)$. Since $A(x) = \int_{-2}^{x} e^t dt$, $A'(x^3) = e^{x^3}$. Thus, $G'(x) = e^{x^3} * (3x^2)$.
  - What is the derivate of $G(x) = \int_{3}^{\sqrt{x}} tan(t) \, dt$
    - $G(x) = A(\sqrt{x}), where\ A(x) = \int_{3}^{x} tan(t)dt$, so $G'(x) = A'(\sqrt{x}) * \left(\frac{1}{2\sqrt{x}}\right)$. Since $A(x) = \int_{3}^{x} tan(t)dt$, $A'(\sqrt{x}) = tan(\sqrt{x})$. Thus, $G'(x) = tan(\sqrt{x}) * \left(\frac{1}{2\sqrt{x}}\right)$.
  - What is the derivate of $G(x) = \int_{0}^{sec(x)} (t^3 - 5t) \, dt$
    - $G(x) = A(sec(x)), where\ A(x) = \int_{0}^{x} (t^3 - 5t)dt$, so $G'(x) = A'(sec(x)) * (sec(x)tan(x))$.
      Since $A(x) = \int_{0}^{x} (t^3 - 5t)dt$, $A'(sec(x)) = (sec(x))^3 - 5sec(x)$. Thus, $G'(x) = (sec(x))^3 - 5sec(x) * (sec(x)tan(x))$.
  - What is the derivate of $G(x) = \int_{1}^{ln(x)} (sin(t)cos(t)) \, dt$
    - $G(x) = A(ln(x)), where\ A(x) = \int_{1}^{x} sin(t)cos(t)dt$, so $G'(x) = A'(ln(x)) * \left(\frac{1}{x}\right)$.
      Since $A(x) = \int_{1}^{x} sin(t)cos(t)dt$,
      $A'(ln(x)) = sin(ln(x))cos(ln(x))$.
      Thus, $G'(x) = sin(ln(x))cos(ln(x)) * \left(\frac{1}{x}\right)$.
  - What is the derivate of $G(x) = \int_{1}^{5^x} (e^t + 3t) \, dt$
    - $G(x) = A(5^x), where\ A(x) = \int_{1}^{x} (e^t + 3t)dt$, so $G'(x) = A'(5^x) * (5^x * ln(5))$.
      Since $A(x) = \int_{1}^{x} (e^t + 3t)dt$,
      $A'(ln(x)) = (e^{5^x} + 3(5^x))$.
      Thus, $G'(x) = (e^{5^x} + 3(5^x)) * (5^x * ln(5))$.

Chapter 5.6

- In chapter 5.6 the classes go over one of the uses that exists for an integral. If you need to calculate the displacement of an object over a period of time, and you know the equation for its rate of change you can find the total displacement from $t_1 to\ t_2$ as equal to the integral from $t_1 to\ t_2$ of the rate of change, or in in mathematics
  - $(position\ at\ t = 1) - (position\ at\ t = 2) = \int_{t_1}^{t_2} v(t)dt$

- If you want the total distance traveled (rather than just the change in position), you need to take the integral of $\int_{t_1}^{t_2} |v(t)| dt$
    - To actually find the absolute value of the function $v(t)$ you need to create a chart like with the 1st derivative test, determine where the function is negative and positive, then split it up into separate integrals. Multiply the integrals over the negative section(s) of the interval to make sure that you get a positive value for each sub-integral.
- Video Resource
    - https://www.youtube.com/watch?v=Gim1ScMnGCE
- Example Problem
    - What was the total displacement of a person moving at velocity $= t^2$ from t=1 to t=3? What is the distance traveled by the person?
        - Displacement $= \int_1^3 t^2 dt = 9 - 1 = 8$
        - Distance $= \int_1^3 |t^2| dt = \int_1^3 t^2 dt = 9 - 1 = 8$
            - Since the function t^2 is always non-negative on the interval [1,3], the displacement and the distance will actually be the same.
    - What was the total displacement of a person moving at velocity $= t^3 - 9t$ from t=1 to t=5? What was the distance?
        - Displacement $= \int_1^5 (t^3 - 9t) dt = (156.25 - 112.5) - (0.25 - 4.5) = 48$
        - Distance $= \int_1^5 |(t^3 - 9t)| dt = -\int_1^3 (t^3 - 9t) dt + \int_3^5 (t^3 - 9t) dt = -((20.25 - 40.5) - (0.25 - 4.5)) + ((156.25 - 112.5) - (20.25 - 40.5)) = 80$
            - The function $t^3 - 9t$ is negative from [1,3], so I had to split the integral into two pieces and manually ensure that both pieces would be positive.
        - Follow up question: in the above example we got a larger distance than displacement. Does this make sense? Is there any time that displacement would be greater than distance?
            - A: It does make sense, and distance is always greater than or equal to displacement.
    - What was the total displacement of a person moving at velocity $= t^2 - 4t$ from t=0 to t=5? What was the distance?
        - Displacement $= \int_1^5 (t^2 - 4t) dt = (\frac{5^3}{3} - \frac{4(5^2)}{2}) - (\frac{1^3}{3} - \frac{4(1^2)}{2}) = -6.67$
            - Does it make sense that we would have a negative displacement?
                - Yes – displacement can be negative. It is distance that is always positive.

- Distance = $\int_1^5 |(t^2 - 4t)|dt = -\int_1^4 (t^2 - 4t)dt + \int_4^5 (t^2 - 4t)dt =$ $-((\frac{4^3}{3} - \frac{4(4^2)}{2}) - (\frac{1^3}{3} - \frac{4(1^2)}{2})) + ((\frac{5^3}{3} - \frac{4(5^2)}{2}) - (\frac{4^3}{3} - \frac{4(4^2)}{2})) = 11.33$

Chapter 5.7

- In chapter 5.7 we are introduced to U substitution as a method for solving integrals. U-sub works by replacing the complicated part of an integral with a variable "u," which we use to make the problem much simpler. Mathematically, we are changing the integral from being $\int f(u(x))u'(x)dx$ to looking like $\int f(u)du$. If our integral is indefinite, we will need to switch back from being in terms of "u" at the end of our problem. On the other hand, if our integral is definite, then we will also need to change the bounds of integration from being "a" and "b" to being "u(a)" and "u(b)."

Another way to write U-substitution

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$

where $u = g(x)$ $du = g'(x)$

Source: https://calcworkshop.com/integrals/u-substitution/

- New Video Resources
  - https://www.youtube.com/watch?v=-NZU-0j6FJ0
  - https://www.youtube.com/watch?v=ipOVrYi_LTE
- Example Problems
  - $\int_1^7 x^2 \sin x^3\, dx$
    - $Set\ u(x) = x^3\ and\ du = 3x^2dx.$
      $As\ such\ we\ can\ rewrite\ the\ integral\ as\ \int_{u(1)}^{u(7)} \sin u * \frac{du}{3}.$
      $\int_{u(1)}^{u(7)} \sin u * \frac{du}{3} = -\frac{1}{3}\cos(u(7)) - (-\frac{1}{3})\cos(u(1)) = -\frac{1}{2}\cos(343) + \frac{1}{3}\cos(1)$
  - $\int_3^9 x\, e^{x^2+1}\, dx$
    - $Set\ u(x) = x^2 + 1\ and\ du = 2xdx.$
      $As\ such\ we\ can\ rewrite\ the\ integral\ as\ \int_{u(3)}^{u(9)} e^u * \frac{du}{2}$
      $\int_{u(3)}^{u(9)} e^u * \frac{du}{2} = \frac{1}{2}e^{u(9)} - \left(\frac{1}{2}\right)e^{u(3)} = \frac{1}{2}e^{82} + \frac{1}{2}e^{10}$
  - $\int_{-5}^7 e^x\, sec^2(e^x)\, dx$
    - $Set\ u(x) = e^x\ and\ du = e^x dx.$
      $As\ such\ we\ can\ rewrite\ the\ integral\ as\ \int_{u(-5)}^{u(7)} sec^2(u) * du$
      $\int_{u(-5)}^{u(7)} sec^2(u) * du = tan(u(7)) - tan(u(-5)) = tan(e^7) - tan(e^{-5})$
  - $\int_0^{10} (x + 1)(x^2 + x - 1)^3 dx$
    - $Set\ u(x) = x^2 + 2x - 1\ and\ du = (2x + 2)dx.$

$$As\ such\ we\ can\ rewrite\ the\ integral\ as\ \int_{u(0)}^{u(10)} u^3 * \frac{du}{2}$$

$$\int_{u(0)}^{u(10)} u^3 * \frac{du}{2} = \frac{1}{4}u(10)^4 - \frac{1}{4}u(0)^4 = \frac{1}{4}(109)^4 - \frac{1}{4}(-1)^4$$

Things Students Tend to Struggle With

- **Using the 2nd part of the Fundamental theorem of Calculus.**
  - Actually applying the 2nd part of the FTC in problems can be really difficult. The problems that require the 2nd part of the FTC tend to be longer and more complicated than your average integration problem. As with all multi-part calculus problems, I recommend trying to break the problem down into its component pieces (in this case A(g(x)) and g(x)), then trying to solve the problem piece by piece, rather than just jumping straight to the final answer.
- **Using the U-Substitution method.**
  - The key to using U-Sub is being able to determine what your "u" is going to be. For the most part this should be relatively straightforward – it will be the "inner" part of a nested function. Also, if you see that one term of the problem is the derivative of another term, the second term will often be the "u" you need to use.