Calculus 1, Week 12

Hey Calculus tutors and students! This resource covers the twelfth week of class. Since I seem to be a bit ahead of the Calculus classes, this resource will mostly be a review of the sections from last week's resource. In addition I will also be covering section 5.4 on the 1st part of the Fundamental Theorem of Calculus.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

Keywords: Riemann Sums, Definite Integrals, Indefinite Integrals.

Key

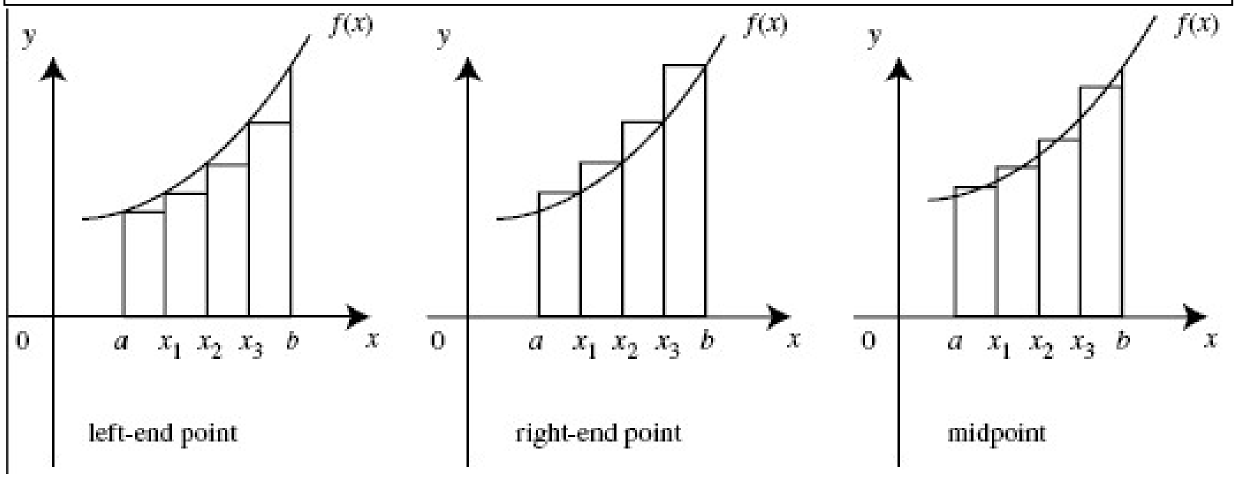
- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

Chapter 5.1 Review

- Example Problems
  - What is the Riemann approximation of the area under $f(x) = Ln(x)$ on the interval [1,7] using a left-hand approximation and three rectangles of equal Δx? Is this an over or under estimation of the actual area under the curve?
    - A: $L_6 = (0 * 2) + (1.0986 * 2) + (1.609 * 2) = 5.4152.$
    - This is an underestimate of the area.
  - What is the Riemann approximation of the area under $f(x) = Ln(x)$ on the interval [1,7] using a left-hand approximation and six rectangles of equal Δx? Is this an over or under estimation of the actual area under the curve?
    - A: $L_6 = (0 * 1) + (0.6931 * 1) + (1.0986 * 1) + (1.3863 * 1) + (1.609 * 1) + (1.7918 * 1) = 6.7618.$
    - This is an underestimate of the area.

The Three kinds of Riemann Sums. Source: Reflections of a Second-career Math Teacher On WordPress https://mathequality.wordpress.com/2013/01/09/building-a-riemann-sum-spreadsheet/



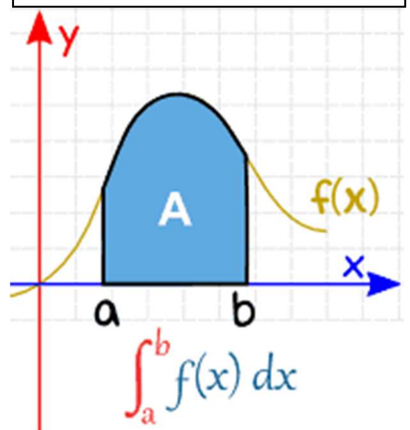
- What is the Riemann approximation of the area under $f(x) = Ln(x)$ on the interval [1,7] using a left-hand approximation and twelve rectangles of equal Δx? Is this an over or under estimation of the actual area under the curve?
  - A: $L_6 = (0 * 0.5) + (0.4055 * 0.5) + (0.6931 * 0.5) + (0.9163 * 0.5) + (1.0986 * 0.5) + (1.2528 * 0.5) + (1.3863 * 0.5) + (1.5041 * 0.5) + (1.609 * 0.5) + (1.705 * 0.5) + (1.7918 * 0.5) + (1.8718 * 0.5) = 7.2087.$
  - This is an underestimate of the area. As you may have noticed, as we continue to increase the number of rectangles our estimation gets closer and closer to the real answer. Since each of our estimations are an underestimation, the result is that our answers are getting higher each time.
  - Follow Up Question: If we did this same exercise of slowly increasing the number of rectangles we use to estimate the area under Ln over this same interval with a right-hand Riemann Sum, what would we expect would happen to our answers?
    - A: We would expect our answer to decrease as we use more rectangles if we were using a right-hand Riemann Sum.

Chapter 5.2 Review

- Example Problems
  - $\int_{-5}^{7} x^{-7} dx$
    - $\frac{(7)^{-6}-(-5)^{-6}}{-6}$
  - $\int_{3}^{10} x^{5/2} dx$
    - $\frac{(10)^{7/2}-(3)^{7/2}}{7/2}$
  - $\int_{1}^{6} x^{3/4} + x^{-1} dx$
    - $\frac{(6)^{7/4}-(1)^{7/4}}{7/4} + Ln(|6|) - Ln(|1|)$
  - Find upper and lower bounds for $\int_{-5}^{5} \sin(x) * cos(x) dx$
    - $\int_{-5}^{5} (-1) dx \leq \int_{-5}^{5} \sin(x) * cos(x) dx \leq \int_{-5}^{5} (1) dx$

$$-10 \leq \int_{-5}^{5} \sin(x) * cos(x) dx \leq 10$$

A Graphic Representation of a Definite Integral



Source: Math Is Fun
https://www.mathsisfun.com/calculus/integration-definite.html

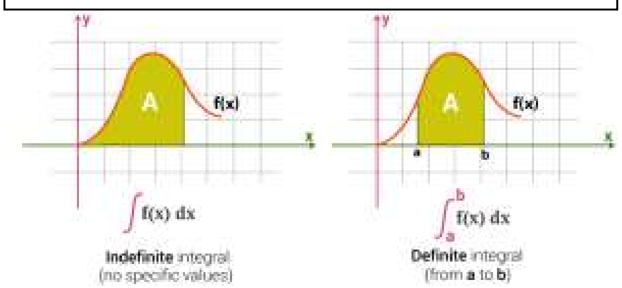
Chapter 5.3 Review

- Example Problems
  - $f(x) = \int (x^{7/4} + x^{-5/6}) dx$
    - $f(x) = \frac{x^{11/4}}{11/4} + \frac{x^{-11/6}}{-11/6} + c$
    - To check our work we can calculate $f'(x)$
      - $f'(x) = (\frac{11}{4}) \frac{x^{7/4}}{11/4} + (-\frac{11}{6}) \frac{x^{-5/6}}{-11/6} + 0$
  - $f(x) = \int (\sin(\frac{1}{7}x) + cos(-7x) dx$
    - $f(x) = -\frac{1}{1/7} \cos\left(\frac{1}{7}x\right) + \frac{1}{-7} \sin(-7x) + c$
    - To check our work we can calculate $f'(x)$
      - $f'(x) = (-7) * \left(-\sin\left(\frac{1}{7}x\right)\right) * (\frac{1}{7}) + (\frac{1}{-7}) * \cos(-7x) * (-7) + 0$
  - $f(x) = \int (\mathrm{e}^{(3/4)\mathrm{x}} + \mathrm{e}^{-11\mathrm{x}}) dx$
    - $f(x) = \frac{1}{3/4} e^{(3/4)x} + \frac{1}{-11} \mathrm{e}^{-11\mathrm{x}} + c$
    - To check our work we can calculate $f'(x)$
      - $f'(x) = (\frac{4}{3}) * e^{(3/4)x} * (\frac{3}{4}) + \frac{1}{-11} \mathrm{e}^{-11\mathrm{x}} * (-11) + 0$

Chapter 5.4

- In chapter 5.4 the students learn the 1st part of the fundamental theorem of calculus, which states that $\int_a^b f(x)dx = F(b) - F(a)$, where F(x) is the anti-derivative of f(x). In addition, the students also learn four new anti-derivatives
  - $\int \sec^2 x\, dx = \tan(x) + c$
  - $\int \csc^2 x\, dx = -\cot(x) + c$
  - $\int \sec(x) \tan(x) dx = \sec(x) + c$
  - $\int \csc(x) \cot(x) dx = -\csc(x) + c$
- Video Resource
  - https://www.youtube.com/watch?v=IYK9angFEbg
- Example Problem
  - $\int_2^4 \sec^2 x\, dx$
    - $\tan(4) - \tan(2)$
  - $\int_{-1}^7 \csc^2 x\, dx$
    - $-\cot(-1) + cot(7)$
  - $\int (\sec(x) \tan(x) + \csc(x) \cot(x)) dx$
    - $\sec(x) - \csc(x) + c$

A Definite Integral is along some interval A to B, while an Indefinite Integral lacks specific endpoints



Source: https://byjus.com/maths/indefinte-integrals/

<u>Things Students Tend to Struggle With</u>

- **1st part of the Fundamental Theorem of calculus**
  - The 1st part of the Fundamental Theorem of Calculus. Looks really strange when set out in functional notation, however the concept is really straight forward. The only thing it says is that a Definite Integral is equal to the anti-derivative at "b" minus the anti-derivative at "a." You have already been using this rule – this is just a formal restatement.