Calculus 1, Week 10

Hey Calculus tutors and students! This resource covers the tenth week of class. Specifically, in this resource I will cover how to sketch its graphs using the 1st and 2nd derivatives test. I will also go over optimization problems.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

Keywords: Sketching a Graph, Optimization.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

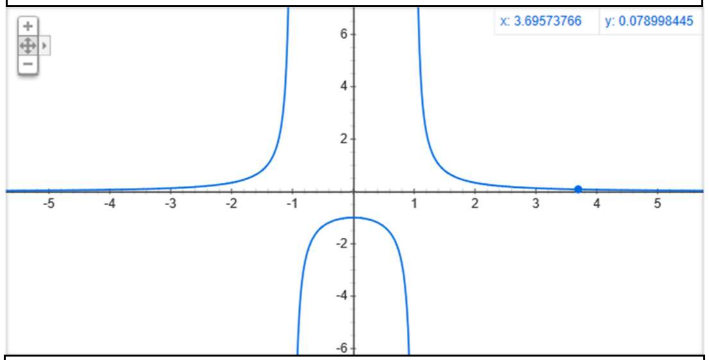
Concepts

Chapter 4.6

- In chapter 4.6 the classes go over how to sketch a graph based on critical points, inflection points, and asymptotes. By looking at vertical asymptotes, end behavior, and the first and second derivatives, we can roughly sketch the graph of even a very complex function.
- Video Resource
  - https://www.youtube.com/watch?v=nplvhffi6Zw&feature=youtu.be
- Example Problem
  - Sketch the graph of $\frac{1}{x^2-1}$
    - Answer: Graph 1
  - Sketch the graph of $\frac{-x^2}{x^2+1}$
    - Answer: Graph 2

- Sketch the graph of $e^x x^4$
  - Answer: Graph 3

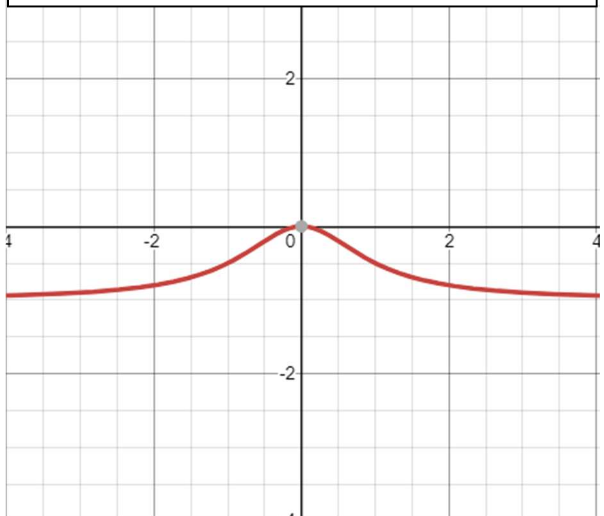
Graph 1



Source: Google Calculator

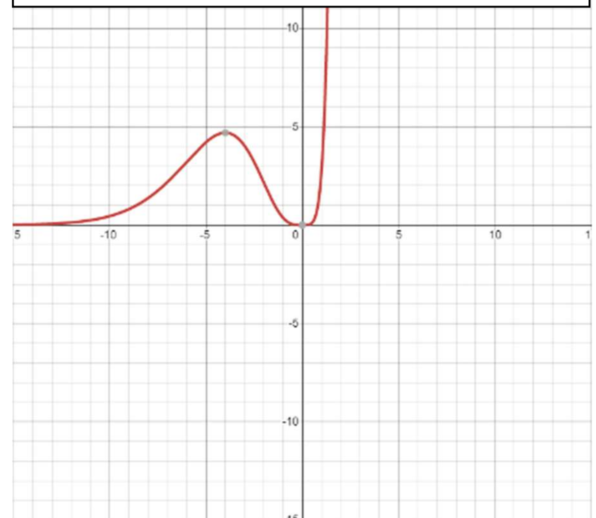
https://www.google.com/search?q=graph+of+1/%28x%5E2-1%29&source=lmns&bih=967&biw=1920&safe=active&hl=en&ved=2ahUKEwjc7ICCo7PoAhWSkZ4KHSOTAbcQ_AUoAHoECAEQAA&safe=active

Graph 2



Source: https://www.desmos.com/calculator

Graph 3



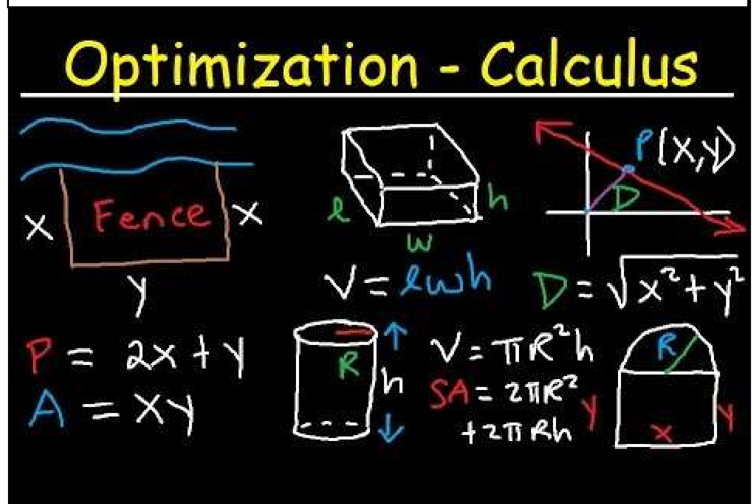
Source: https://www.desmos.com/calculator

Chapter 4.7

- In chapter 4.7 we are introduced to optimization using derivatives. Now, optimization in theory is no different than finding a local maximum or minimum in a function. The big difference is that before when we were finding the local max or min of a function in terms of just one variable, but now our functions seem to be in terms of two or more variables. Since we don’t know how to take derivatives in terms of more than one variable, what we need to do is convert our function from being in terms of two variables to being in terms of just one variable.

- Usually an optimization problem starts with two equations. The first relates what we want to optimize (surface area, for instance) to our independent variables (length, width, and height of a box, for instance). The second relates our independent variables to each other. Please note, you may need to infer or come up with one or both equations on your own, like back with related rates. We can solve our second equation for one of the independent variables, then plug that into our first equation. We should be left with a function in terms of only one variable which we can find maxes and mins for just like in section 4.6.

Some very common shapes used in optimization problems

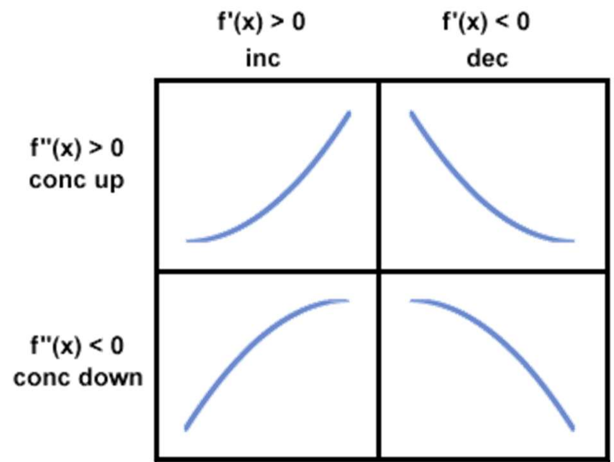


Source: The Organic Chemistry Tutor YouTube channel

- Video Resource
  - https://www.youtube.com/watch?v=X6fEBmyL-18&feature=youtu.be
  - https://www.youtube.com/watch?v=pi40o4Cuquw&feature=youtu.be
- Example Problem
  - Minimize the amount of fencing needed to enclose a pasture with an area of $500yards^2$ given the pasture is bordered by a stream to the north, so that side doesn't need a fence. What are the dimensions of the pasture?
    - Our first equation for this question will be: $Perimeter = 2y + x$
    - Our second equation will be: $500 = xy$
    - A: $x \approx 31.623yards, \quad y \approx 15.811yards$
  - Minimize the surface area of an open-topped box with a volume of $100cm^3$ given that the base of a box is a square. What is the height of the box?
    - Our first equation for this question will be: $Surface\ Area = s^2 + 4sh$
    - Our second equation will be: $100 = s^2h$
    - A: $h \approx 2.92cm$
  - Minimize the cost to produce an open-topped box with a volume of $100cm^3$ given that each square cm of material on the sides costs \$0.50, each square cm of material on the bottom costs \$1.50, and the base of a box is a square. What is the height of the box?
    - Our first equation for this question will be: $Cost = 1.50s^2 + 0.50(4sh)$
    - Our second equation will be: $100 = s^2h$
    - A: $h \approx 6.083cm$

Things Students Tend to Struggle With

- **Sketching the graph of a function from its derivative.**
    - When sketching a graph of a derivative I usually begin by doing the 1st and 2nd derivative tests. After I have the line charts that describe when the function is increasing/decreasing and concave up/concave down I look at end behavior by taking the limit of the function as x goes to positive and negative infinity. From here the only tricky part is remembering how the different combinations of slope and concavity look on a graph. To the right is a handy chart to help with remembering.



Source: SparkNotes

https://www.sparknotes.com/math/calcab/applicationsofthederivati

- **Optimization Problems**
    - At their core, optimization problems are just the same as finding maximums and minimums on a function. There isn't a real difference between finding the lowest point on a function and minimizing the cost of fencing for your pasture. The twist is that previously you have been given the function you are trying to find the maximums and minimums for, but now you are going to need to produce them yourself. Personally, I always draw a picture to help me out. Once you have the function for the value you are trying to maximize or minimize (surface area, cost, etc.) you can use the 1st and 2nd derivative tests to solve the problem as before.