MTH 1322: Calculus II
Week 9 Tutoring Resources

Ethan Reyes

March 15th, 2021

Welcome Calculus II tutors and students! In this week's resource we will be reviewing shell method as well as working problems with work and energy. For more help with these topics please schedule a 1-on-1 visit with me or another tutor. **Please visit baylor.edu/tutoring to make an appointment and to reserve a spot for the Calculus II group tutoring session every Tuesday at 6:30pm.** If you would like to view any of the previous resources please click **HERE**.

Overview[1]

KEYWORDS: Shell Method / Hooke's Law / Work and Energy

1 New Topics

1.1 Shell Method

In Previous resources we covered shell method, but since there are some sections that have just covered this topic we will work some more examples. Recall the volume of a solid rotated about an axis using shell method uses the following:

$$V = 2\pi \int_a^b (\text{radius})(\text{height of shell})dx \tag{1}$$

Let's work an example. Suppose we are asked to find the volume of obtained by rotating the region enclosed by the following equations about the y-axis:

$$f(x) = x^2 \quad g(x) = \sqrt{x} \tag{2}$$

If we were asked to compute this integral without looking at the graph our first step would be to set the two equation equal to one another to find where the two equations intersect. By inspection we find that for $x = 0$ and $x = 1$ are the two points where the equations intersect. Next we need to determine which function is greater than the other, in other words we need to determine if $f(x) \leq g(x)$ or if $g(x) \leq f(x)$. To do so we will pick a point between $x = 0$ and $x = 1$. If we choose $x = .5$ we find that $f(.5) = .5^2 = .25$ and $g(.5) = \sqrt{.5} = .70710$. Therefore we can determine that $f(x) \leq g(x)$. Our next step would be to then set up integral and then evaluate.

Suppose we are allowed to graph the two functions, we would find exactly the same results by careful inspection. The shaded region below is the region we are rotating about the y-axis.

[1] The information used to create this resource was taken from this source: [1]

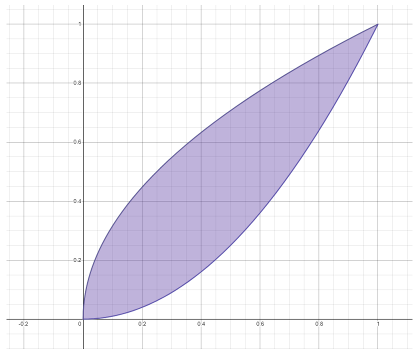

Figure 1:

Since we can see that the two equations intersect at $x = 0$ and $x = 1$ we now have our bounds of integration. Shell method tells us that the radius of the shell we are integrating in this case is x. Similarly, we can describe the height of the shell by subtracting $g(x) - f(x)$ and therefore we have the following integral:

$$\int_0^1 x(\sqrt{x} - x^2)dx \tag{3}$$

Now we can multiply through by x to simplify to get:

$$\int_0^1 x^{3/2} - x^3 dx \tag{4}$$

Therefore by basic integration rules we find that the solution to our problem is: 0.15. If you would like to watch a short video about solving integrals using shell method please click **HERE** [2].

1.2 Work and Energy

When working with problems involving work and energy we need to be cautious of certain things. For example, the work done on an object by applying constant force is found by the equation: $W = F \cdot d$, where F is the force and d is the distance the object is moved. For example, suppose we are asked to find the work done on an object that has moved $3m$ by applying a constant force of $22N$. If we apply the equation above we can express the work done by: $W = 3 \cdot 22 = 66N \cdot m$.

However, the work done on object by applying variable force is found by: $W = \int_a^b F(x)\,dx$ where $F(x)$ is the equation describing the force applied on the object. For example, suppose we are asked to find the work done on an object that moved from $x = 2m$ to $x = 5m$ where the force applied to the object is expressed by:

$$F(x) = x^{3/2} + x^2 - 4x + \sqrt{x-2} + 13 \tag{5}$$

To find the work done on the object we need to integrate $F(x)$:

$$\int_2^5 x^{3/2} + x^2 - 4x + \sqrt{x-2} + 13\,dx \tag{6}$$

Now we can integrate using basic integration techniques.

$$\frac{2}{5}x^{5/2} + \frac{1}{3}x^3 - 2x^2 + \frac{2}{3}(x-2)^{3/2} + 13x\Big|_2^5 \tag{7}$$

After using a calculator to compute the solution we find the answer is 59.562 $N \cdot m$. Therefore the final answer to our original problem is:

$$\int_2^5 (x^{3/2} + x^2 - 4x + \sqrt{x-2} + 13)\,dx = 59.562 \tag{8}$$

It is important to note that is possible we may need to use some of the previously learned integration techniques such integration by parts or partial fractions etc. In this case we do not need to worry but it is still a good idea to practice these methods by hand. Another application of variable force are spring related problems. Finally, the last important formula to remember is the following:

$$W = \int_a^b L(y)\,dy \tag{9}$$

where $L(Y) = g\times$ density $\times A(y)\times$ (vertical distance lifted).

Let's work an example. Suppose we are asked to calculate the work required to lift a 3-m chain over the side of a building if the chain has variable density of $\rho(x) = x^2 - 3x + 10$ kg/m for $0 \leq x \leq 3$. From the start we can realize that the bounds of integration are from $x = 0$ to $x = 3$. Since we are not given $A(y)$ we may assume this value is equal to one. Additionally, we also know that g represents gravity so we can let $g = 9.8$. Thus, we can see we are only missing the vertical distance the object is lifted. To determine this value we need to think about what the problem is asking. The problem does not specify the height lifted, but notice the height depends on x therefore we can say the height at any point is x. In other words we can write the following integral:

$$W = \int_0^3 (9.8)(x^2 - 3x + 10)(x)\,dx \tag{10}$$

Since 9.8 is a constant we can pull it out of the integral. Additionally, to simplify we can multiply through by x to get:

$$W = 9.8\int_0^3 x^3 - 3x^2 + 10x\,dx \tag{11}$$

Using basic integration techniques we find the solution to be 92.25 $N \cdot m$. If you would like to watch a short video about solving work and energy problems please click **HERE** [3].

References

[1] J. Rogawski, C. Adams, and R. Franzosa, *Calculus: Early Transcendentals*, 4th ed. New York: W. H. Freeman, Dec. 2018.

[2] The Organic Chemistry Tutor, "Shell Method - Volume of Revolution," Jan. 2021. [Online]. Available: https://www.youtube.com/watch?v=D5sT1br9soI

[3] blackpenredpen, "Calculus Sect 6 4 #21, Calculating Work," Apr. 2015. [Online]. Available: https://www.youtube.com/watch?v=-DEkriNuPNQ