Calculus 1, Week 6

Hey Calculus tutors and students! This resource covers the sixth week of class. Specifically, in this resource I will cover how to use the Chain Rule to take derivatives, how to use Implicit Differentiation, how to take derivatives of exponential and logarithmic functions, and how to take derivatives of hyperbolic trig functions.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

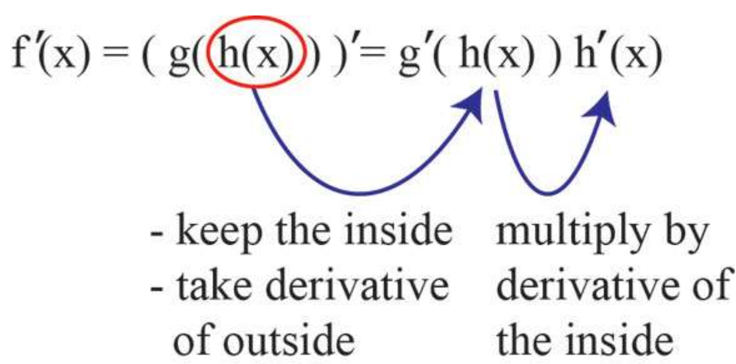
Keywords: Chain Rule, Implicit Differentiation, Derivatives of Logarithmic Functions, Derivatives of Exponential Functions, Derivatives of Hyperbolic Trig Functions.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

- Chapter 3.7
  - Chapter 3.7 is where calculus students are introduced to the dreaded chain rule. As you may remember, the chain rule states that $\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'(g(x))g'(x)$. People always forget to include the g'(x), so always be sure to include it.
  - Example Problem
    - What is $\frac{d}{dx}(\sin(x))^2$?
      - $2(\sin(x)) * \cos(x) * \frac{dx}{dx}$

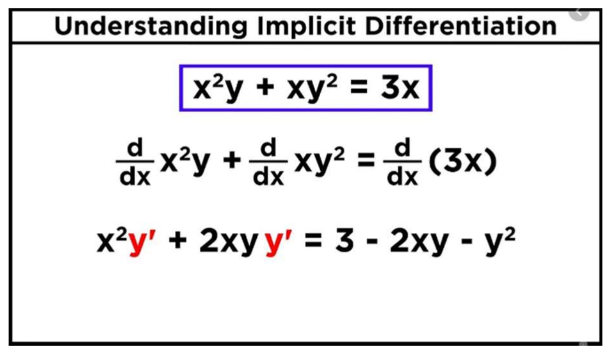


Source: https://www.mathbootcamps.com/the-chain-rule/

- Remember, the chain rule always applies, its just that the "inside" of a simple function like sin(x) is just "x", and $\frac{d}{dx}(x) = \frac{dx}{dx} = 1$, so the rest of the equation is unaffected.

- What is $\frac{d}{dx}\left(\sec\left(\frac{1}{x^2}\right)\right)$?
  - $\sec\left(\frac{1}{x^2}\right) tan\left(\frac{1}{x^2}\right) * -2x^{-3} * \frac{dx}{dx}$
- What is $\frac{d}{dx}\left(\ln\left(sin\left(\frac{3x}{e^x}\right)\right)\right)$?
  - $\frac{1}{\left(sin\left(\frac{3x}{e^x}\right)\right)} * cos\left(\frac{3x}{e^x}\right) * \left(\frac{3e^x - 3xe^x}{e^{2x}}\right)$

- Chapter 3.8
  - In chapter 3.8 we learn about implicit differentiation. Also in this chapter the book introduces the derivatives of Arcsine and Arccosine, as well as the other inverse Trig functions. Their derivatives are as follows
    - $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
    - $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
    - $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2+1}$
    - $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$
    - $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{x^2+1}$
    - $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$



Source: https://www.youtube.com/watch?v=M0SMSWM2oZA

  - Video Resources
    - https://www.khanacademy.org/math/ap-calculus-ab/ab-differentiation-2-new/ab-3-2/v/implicit-differentiation-1
  - Example Problem
    - Solve for $\frac{dy}{dx}$ $if\ y^3 + e^x = 27$
      - $\frac{dy}{dx} = \frac{-e^x}{3y^2}$
    - Solve for $\frac{dy}{dx}$ $if\ ln(y^3) + sin\left(\frac{1}{x^2}\right) = \sin^{-1}(5x)$
      - $\frac{dy}{dx} = \left(\frac{y}{3}\right)\left(\frac{5}{\sqrt{1-25x^2}} - cos\left(\frac{1}{x^2}\right)(2x)\right)$
    - What is the derivative of arcsin x^2
      - $\frac{1}{\sqrt{1-x^4}} * 2x$

- Chapter 3.9
  - In this chapter we are introduced to the general derivatives of logarithmic and exponential functions. The general forms of their derivatives are as follows
    - $\frac{d}{dx}b^x = \ln(b) * b^x$
    - $\frac{d}{dx}\log_b x = \frac{1}{\ln(b)*x}$

- Also in this section, we are introduced to the hyperbolic functions sinh(x) and cosh(x). Their definitions and derivatives are as follows
    - $\sinh(x) = \frac{e^x - e^{-x}}{2}$
    - $\cosh(x) = \frac{e^x + e^{-x}}{2}$
    - $\frac{d}{dx}\sinh(x) = \cosh(x)$
    - $\frac{d}{dx}\cosh(x) = \sinh(x)$
- Example Problems
    - What is the derivative of $\log_5 x^2$?
        - $\frac{2}{\ln(5) * x}$
    - What is the derivative of $\cosh(5^{3x})$?
        - $\sinh(5^{3x}) * (\ln(5) * 5^{3x}) * (3)$
    - Show that $\frac{d}{dx}\sinh(x) = \cosh(x)$

Things Students Tend to Struggle With

- **Chain Rule**
    - The Chain Rule is one of the most difficult concepts in Calculus 1 for students to understand. Perhaps the single most common question I have gotten as a tutor is "when does the Chain Rule apply." The answer is that the Chain Rule **ALWAYS** applies. We have been ignoring the fact that it always applies up until now because often the "inside" of a simple function like sin(x) is just "x", and $\frac{d}{dx}(x) = \frac{dx}{dx} = 1$. However, even in this situation the Chain Rule is occurring – we are just doing that step in our head.
- **Implicit Differentiation**
    - Implicit Differentiation – at its core – is a really simple concept. Whereas before we were more-or-less only working with the right hand side of an equation when we take a derivative, now we have to do all the same steps for the left hand side. The one other wrinkle is that unlike "x", "y" doesn't disappear when you take a derivative. While $\frac{d}{dx}(x) = \frac{dx}{dx} = 1$, in the case of "y" we get $\frac{d}{dx}(y) = \frac{dy}{dx} = y'$.