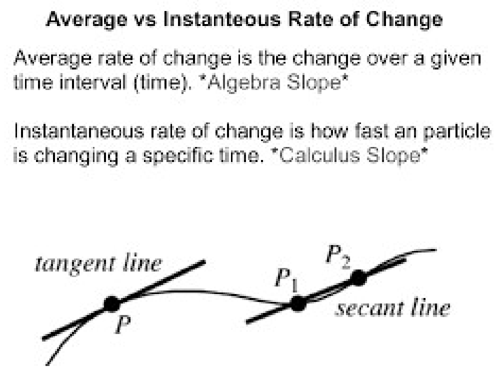
Calculus 1, Week 4

Hey Calculus tutors and students! This resource covers the fourth week of class. Specifically, in this resource I will cover how to calculate derivatives using their limit definition, as well as the Product and Quotient Rules.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson



Source: https://mcdowellakmath.weebly.com/uploads/2/4/5/4/24545915/1_velocity_and_rates_of_change.pdf

Keywords: Limits, Derivatives, Product Rule, Quotient Rule.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

- **3.1: Definition of the Derivative (Review)**
  - Because chapters 3.1 and 3.2 are so interrelated, I decided to include a review of 3.1 in this resource. If you want to see more on this chapter, check out my week 3 resource as well.
  - Worked Example
    - In general, I don't like doing worked examples in my resource, since I find that our YouTube Videos do a much better job of talking through examples step by step. However, it may be helpful here to see an example problem directly worked through.
    - Find the derivative of $f(x) = \sqrt{x^2 - 1}\ at\ the\ point\ x = 5$
      - From the limit definition of the derivative we know that

$$f'(x) = \lim_{h->0} \frac{f(x+h) - f(x)}{h} = \lim_{h->0} \frac{\sqrt{(x+h)^2 - 1} - \sqrt{x^2 - 1}}{h}$$

      - When using this version of the Limit Definition of the Derivative, our first goal is to manipulate the equation until we can pull an "h" out of the entirety of the numerator of the equation. For this equation, that means simplifying the numerator by multiplying it by its Conjugate.

$$f'(x) = \lim_{h->0} \frac{\sqrt{(x+h)^2-1} - \sqrt{x^2-1}}{h}$$

$$= \lim_{h->0} \frac{\sqrt{(x+h)^2-1} - \sqrt{x^2-1}}{h} * \frac{\sqrt{(x+h)^2-1} + \sqrt{x^2-1}}{\sqrt{(x+h)^2-1} + \sqrt{x^2-1}}$$

- Once we simplify, we are left with

$$f'(x) = \lim_{h->0} \frac{x^2+2xh+h^2-1-x^2+1}{(h)(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})} = \lim_{h->0} \frac{2xh+h^2}{(h)(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})}$$

- Now that we have an "h" that we can pull out of the numerator we can pull it out and cancel our "h" in the denominator.

$$f'(x) = \lim_{h->0} \frac{h(2x+h)}{(h)(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})} = \lim_{h->0} \frac{(2x+h)}{(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})}$$

- Now that we have canceled the "h" that was in our denominator, we should be able to evaluate the limit by plugging in 0 for all our remaining "h's"

$$f'(x) = \frac{(2x+0)}{(\sqrt{(x+0)^2-1} + \sqrt{x^2-1})} = \frac{2x}{2\sqrt{x^2-1}} = \frac{x}{\sqrt{x^2-1}}$$

- Now all we have to do is plug in x=5 to get our answer

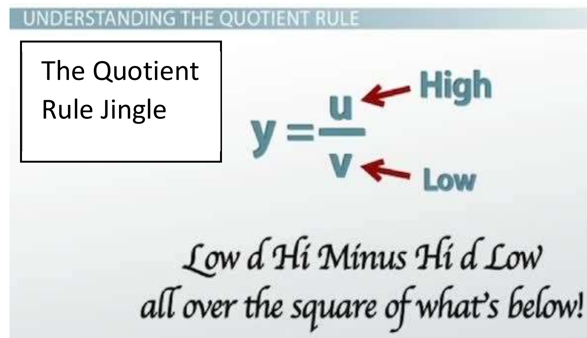
$$f'(5) = \frac{5}{\sqrt{24}}$$

- Examples
  - What is the derivative of f(x) = x^2 at x = 3?
    - A: $f'(3) = 6$
  - Find the Derivative of $f(x) = 3x^2 + 7x + 10$ at $x = 4$
    - A: $f'(4) = 31$

- **3.2: The Derivative as a Function**
  - Where in Ch. 3.1 we computed the derivative of a function at a specific point, now we want to create a function that describes the derivate of f(x) at any point. The method is going to be very similar, but we won't actually plug in any specific value of x.
  - Video
    - https://www.youtube.com/watch?v=EyN92I3jk1k
  - Examples

- What is the derivative of f(x) = x^2?
  - A: $f\ '(x) = 2x$
- Find the Derivative of $f(x) = 3x^2 + 7x + 10$
  - A: $f\ '(x) = 6x + 7$

- **3.3: Product and Quotient Rules**
  - In chapter 3.3 we are introduced to two of the most helpful rules of differentiation in Calc 1: the product and the Quotient Rules. Mathematically, we write these two rules like this.
    - Product Rule: $\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + g'(x)f(x)$.
    - Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g\prime(x)}{g(x)^2}$.
    - It can be very helpful to try and memorize these rules using a jingle, such as the one I have provided to the right.
  - Video Resources
    - https://www.youtube.com/watch?v=kxqmLULgiTI
  - Example Problem
    - What is the derivative of f(x) = $\frac{x^3}{lnx}$?
      - f'(x) = $\frac{lnx*3x^2 - x^3*\frac{1}{x}}{(lnx)^2}$



Source: https://study.com/academy/lesson/when-to-use-the-quotient-rule-for-differentiation.html

<u>**Things Students Tend to Struggle With**</u>

- **Limit Definition of the Derivative**
  - When using the $f\ '(x) = \lim_{h->0} \frac{f(x+h)-f(x)}{h}$ version of the limit definition of the derivative, the whole goal is to pull an "h" out of the numerator to cancel with the one in the denominator. For $f\ '(x) = \lim_{x->a} \frac{f(x)-f(a)}{x-a}$ the same is true, except what you really need to cancel is the "$x - a$" term.
- **Product Rule**
  - It is often easy to forget to apply the Product Rule when taking derivatives – especially on longer problems that have many layers. In general, the best way to avoid these kinds of silly mistakes is to take your time and be methodical while working through problems.
- **Quotient Rule**
  - All of the same issues with the Product Rule also apply for the Quotient Rule, but making everything more difficult is the fact that the formula for the Quotient rule is much more complicated. In general, I find it very helpful to try and memorize the Quotient Rule by using a jingle or some other memorization technique.