MTH 1320 – PRECALCULUS
FALL 2020 WEEK 8 RESOURCE

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Welcome back, Precalculus scholars! This resource focuses on material covered in the eighth week of classes, namely exponential functions from Sections 4.1-4.2 of OpenStax's *Precalculus*. Please refer to previous resources if you would like to review other topics. Don't forget to sign up for Group Tutoring on Tuesdays at 5:00! Good luck to those who have tests this week!

4.1 Exponential Functions

Exponential functions model outputs that change at a rate proportional to the current quantity and have the form

$$f(x) = ab^x,$$

where a is the **initial value** ($f(0)$) and b is the **base** or **growth factor**. Some restrictions on a and b are:

- a must not be equal to zero. ($a = 0$ would result in the constant function $f(x) = 0$.)
- b must be positive. ($b < 0$ would result in a complex, oscillating output.)
- b must not be equal to 1. ($b = 1$ would result in the constant function $f(x) = a$.)
- a and b must be real numbers.

Some characteristics of an exponential function are:

- Its domain is $(-\infty, \infty)$.
- Its range is
  - $(0, \infty)$ if $a > 0$.
  - $(-\infty, 0)$ if $a < 0$.
- Its y-intercept is $(0, a)$.
- It has a horizontal asymptote at $y = 0$.

When evaluating an exponential function, make sure that you raise b to the input value x before multiplying by a in order to follow the order of operations.

An output that grows exponentially increases by the same percentage for equal changes in the input. This corresponds to multiplying the output by a constant every time the input increases by a certain amount. For example, when we have the function $f(x) = 3^x$, the output triples (or increases by 300%) every time the input increases by 1. See Table 1.

Table 1. $f(x) = 3^x$

Input, x	Output, $f(x) = 3^x$	Relationship with Previous Output
0	1	
1	3	$1 \times \mathbf{3}$
2	9	$3 \times \mathbf{3}$
3	27	$9 \times \mathbf{3}$
4	81	$27 \times \mathbf{3}$

Writing Equations for Exponential Functions

As with writing the equation for a linear function, the process of writing the equation for an exponential function changes based on the values given in the problem statement.

If they are given, you can simply substitute the initial value a and growth factor b into the form $f(x) = ab^x$. Note that if the given information says that the output increase by a certain percentage p, the base is $b = 1 + p$. For example, a population increase by 19% every year would correspond to the base $b = 1 + 0.19 = 1.19$.

If instead two points (x_1, y_1) and (x_2, y_2) are provided, substitute each point into the form $f(x) = ab^x$ to obtain the two equations $y_1 = ab^{x_1}$ and $y_2 = ab^{x_2}$. Then, solve the system of equations for a and b and substitute them into the form $f(x) = ab^x$.

Note that if you are given a or b and a point, you can use a blend of the above approaches. Additionally, you can extract two points from a graph of to write the equation of an exponential function.

The Number e and the Natural Exponential

There is one base that is used very frequently in mathematics denoted by the letter e. This number is defined as follows:

$$\text{as } n \to \infty, \qquad \left(1 + \frac{1}{n}\right)^n \to e.$$

e is approximately equal to 2.72. If you ever see lowercase e in a function, remember that it is just a constant!

The function $f(x) = e^x$ is called the **natural exponential**.

Formulas Involving Exponentials

Below are some exponential formulas that you should be familiar with.

THE COMPOUND INTEREST FORMULA

Compound interest can be calculated using the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

where

- $A(t)$ is the account value,
- t is measured in years,
- P is the starting amount of the account, often called the principal, or more generally present value,
- r is the annual percentage rate (APR) expressed as a decimal, and
- n is the number of compounding periods in one year.

Figure 1. Definition of the Compound Interest Formula
Source: [1]

In the above formula, if we let the number of compounding periods per year n approach infinity, we get the **continuous compounding formula**

$$A(t) = Pe^{rt}.$$

Note that **continuous growth** or **continuous decay** models are “exponential models that use e as the base” [1]. A generalization of the continuous compounding formula can be used to model any continuous growth or decay. The **continuous growth/decay formula** is

$$A(t) = ae^{rt}.$$

Just like for general exponential functions, a is the initial value. r is the continuous growth rate per unit time, and t is the elapsed time.

4.2 Graphs of Exponential Functions

Graphs of exponential functions with $b > 0$ look like the curves in Figure 2.

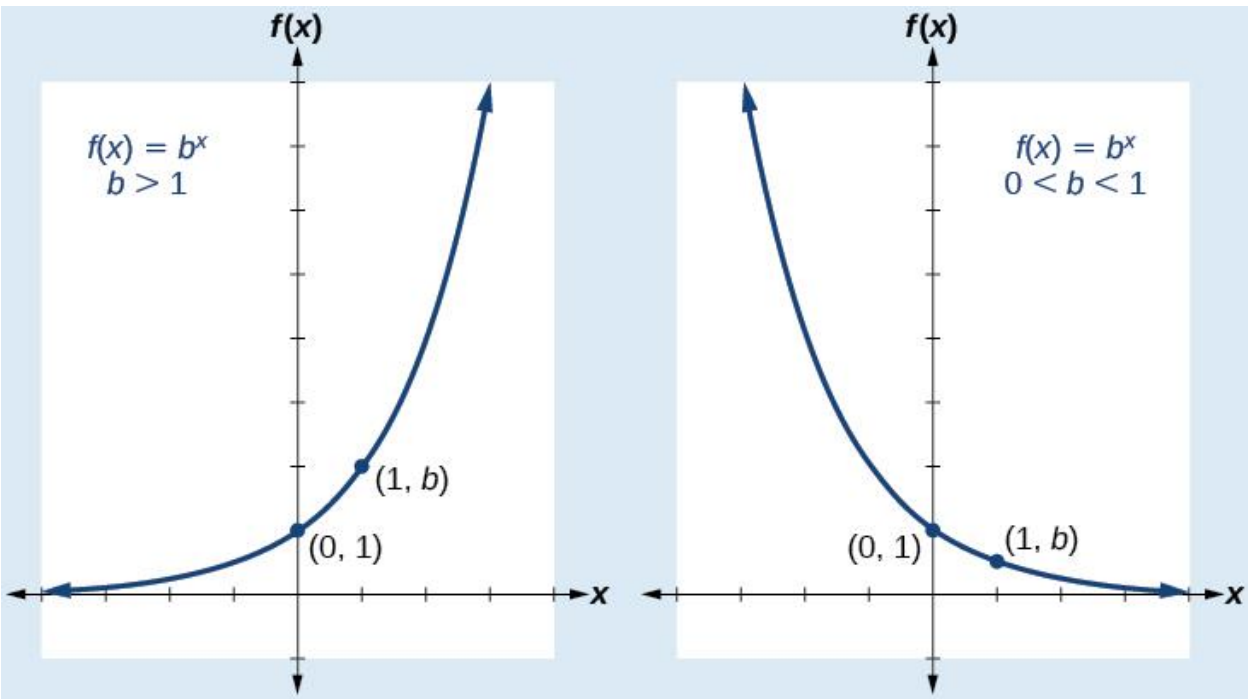

Figure 2. The Parent Exponential Function $f(x) = b^x$
Source: [1]

To graph an exponential function of the form $f(x) = b^x$, you can follow these steps:

1. Evaluate the function at three x values to obtain three points. Make sure to include $x = 0$.
2. Plot the three points on a coordinate plane.
3. Sketch a curve through the points.

Transformations of exponential functions can be handled similarly to transformations of any other function. (See the Week 3 Resource for a summary of transformations of functions.) Note that vertical shifts like $g(x) = b^x + c$ move the horizontal asymptote from $y = 0$ to $y = c$.

Reminders

Thank you for reading this resource! Here are a few reminders about other Precalculus opportunities:

- Group Tutoring for Precalculus is on Tuesdays from 5:00 to 6:00 p.m. through Microsoft Teams. If you have not received an email inviting you to sign up, and you want to attend, please email me at Sydney_Schirner1@baylor.edu, and I will make sure you are included!
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/support_programs/index.php?id=955694.
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at https://www.baylor.edu/tutoring.

References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: https://openstax.org/details/books/precalculus