MTH 1320 – PRECALCULUS
FALL 2020 WEEK 4 RESOURCE

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Welcome back, Precalculus scholars! This resource focuses on material covered in the fourth week of classes, namely topics from sections 2.1-3.2 of OpenStax's *Precalculus*. Don't forget to sign up for Group Tutoring on Tuesdays at 5:00! Good luck to those who are having their first test this week!

2.1 Linear Functions

A **linear function** is "a function with a constant rate of change, that is, a polynomial of degree 1" [1]. For example, $f(x) = 5x + 4$ is a linear function because the highest power of x it has is x^1 or just x. The graph of a linear function is always a straight line and therefore can be written in **slope-intercept form**:

$$f(x) = mx + b$$

where m is its **slope** or **rate of change** and b is its ***y*-intercept**, the value at which it crosses the *y*-axis.

The slope of a linear function determines whether it is increasing, decreasing, or constant. For **increasing** functions, the outputs increase with the inputs, and the graph has a positive slope (see Figure 1(a)). For **decreasing** functions, the outputs decrease as the inputs increase, and the graph has a negative slope (see Figure 1(b)). A **constant** function ($f(x) = c$) outputs only one value, no matter the input. Its graph is therefore a horizontal line with a slope of zero (see Figure 1(c)).

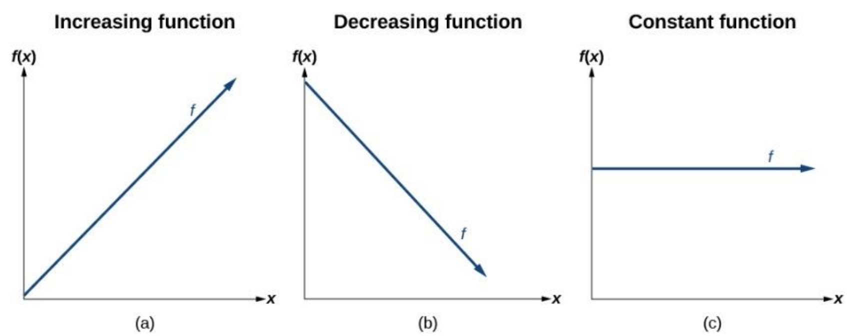

Figure 1. Different kinds of linear functions
Source: [1]

Given two points (x_1, y_1) and (x_2, y_2) from a linear function, its slope m can be calculated using

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1},$$

where Δ stands for "change in." The change in y is also know as the **rise**, and the change in x is also known as the **run**.

When given two points from a linear function, it is often convenient to write it in **point-slope form**:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is one of the given points and m is again the slope.

2.2 Graphs of Linear Functions

Graphing Methods

There are three methods, given in OpenStax's *Precalculus*, that you can use to graph a linear function.

Method 1: Plotting Points

To graph a linear function by plotting points,

1. Choose two (or more) input values (ideally, integer inputs that will give you integer outputs).
2. Evaluate the function at each input value to find the corresponding output value.
3. Plot each input and its output as a point (x, y) on a coordinate plane (grid).
4. Draw a straight line through the points.

Method 2: Using the Slope and y-intercept

If a linear function is given in slope-intercept form ($f(x) = mx + b$), it is convenient to use this method:

1. Identify the *y*-intercept (b) of the function.
2. Plot the point $(0, b)$ on a coordinate plane.
3. Identify the slope (m) of the function.
4. Determine the rise (change in y) and run (change in x), using the formula $m = \frac{rise}{run}$. Note that if the slope is an integer, the run is 1.
5. Starting from the point $(0, b)$, move up by $rise$ units and move right by run units. See Figure 2. (If your slope is negative, move down by $|rise|$ units.)
6. Plot a point at your new location.
7. Continue to "rise," "run," and plot a few more points.
8. Draw a straight line through the points.

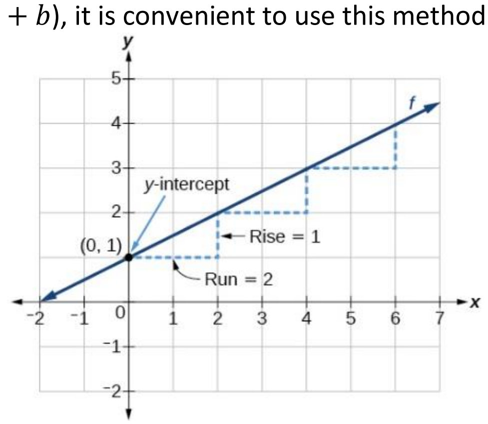

Figure 2. How to Rise and Run
Source: [1]

Note that you can use a similar method if the linear function is written in point-slope form ($y - y_1 = m(x - x_1)$), using the given point (x_1, y_1) as your starting point instead of the *y*-intercept.

Method 3: Transforming the Identity Function

An alternative method when the function $f(x)$ is given in slope-intercept form is using transformations.

1. Graph the identity function, $y = x$.
2. Vertically stretch/compress the graph of $y = x$ based on the slope m of $f(x)$.
3. Vertically shift the graph of $y = mx$ by b units, where b is the *y*-intercept of $f(x)$.

The *x*-intercept

The ***x*-intercept** of a function is where its graph crosses the *x*-axis. To find this *x*-value, set the function equal to zero, and solve for x.

Horizontal and Vertical Lines

A **horizontal line** is the graph of a constant function (see Figure 1(c)). It is written in the form $f(x) = b$, where b is the function's *y*-intercept and only output. The mx term is missing, so the slope m is zero.

A **vertical line** has only one input value and infinite output values. This means that a vertical line is not a function. The equation of a vertical line is written in the form $x = a$, where a is its *x*-intercept. Because the change in x of the graph is zero, the slope of a vertical line is undefined.

Parallel and Perpendicular Lines

For this section, let's establish that we have two lines: $f_1(x) = m_1x + b_1$ and $f_2(x) = m_2x + b_2$.

The lines are **parallel** if they never intersect. See Figure 3. In order to not intersect, they must have the same slope, i.e. $m_1 = m_2$. If the lines also have the same *y*-intercept, the lines intersect at *every* point and are called **coincident**. In other words, if $m_1 = m_2$ and $b_1 = b_2$, $f_1(x)$ and $f_2(x)$ are the same line!

The lines are **perpendicular** (or orthogonal) if they intersect at an angle of 90°. See Figure 4. This intersection occurs if $m_2 = -\frac{1}{m_1}$, or the slopes are the "negative reciprocal" of each other.

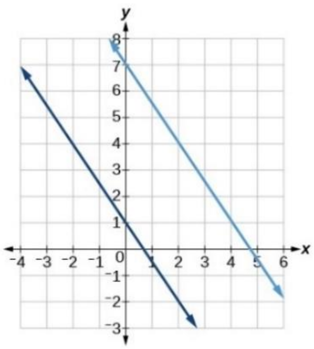

Figure 3. Parallel Lines
Source: [1]

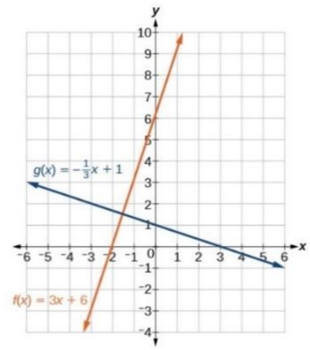

Figure 4. Perpendicular Lines
Source: [1]

The Solution to a System of Two Linear Equations

A system of equations is a set of two or more equations. The solution to that system is the point(s) where the graphs of the equations intersect. For a system of two linear equations with different slopes, the solution is one point. To find this one point,

1. Set the two functions equal to each other.
2. Solve for x.
3. Evaluate one of the functions at that x-value to determine the corresponding y-value.

Section 2.3 Modeling with Linear Functions

It is an important skill to model real-world situations with linear functions. We must adapt our modeling method based on the information we have. Remember that the independent variable is the input, and the dependent variable is the output. If there is an initial condition given, it is the function's *y*-intercept. Look for words like "per," "every," or "each" to find the function's rate of change. Sometimes, instead of the slope and *y*-intercept, we are given sets of an input and output. Each set is a coordinate pair, and we can use two pairs to calculate the slope. Also, if time is one of the variables, it is often the input.

Note that it is often helpful to draw a diagram for problems that involve geometric relationships.

3.2 Quadratic Functions

A quadratic function is "a function of degree two" [1]. Its graph is a parabola. For example, the toolkit function $f(x) = x^2$ is the basic quadratic function. The **general form** of a quadratic function is

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are real constants and $a \neq 0$. (If a were zero, the function would not be of degree 2.)

It is often easier to visualize a parabola when its equation is in **standard form**:

$$f(x) = a(x-h)^2 + k,$$

where (h, k) is the **vertex**, or lowest/highest point, of the parabola (see Figure 5). If $a > 0$, the parabola opens upward (like a smile), and if $a < 0$, the parabola opens downward (like a frown).

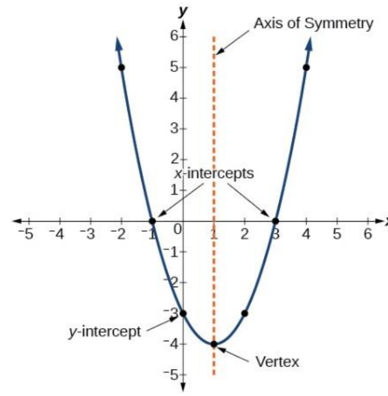

Figure 5. The Features of a Parabola
Source: [1]

To determine the vertex of a parabola from the general form of the quadratic function,

1. Calculate the *x*-coordinate of the vertex using the formula
$$h = -\frac{b}{2a}$$
2. Evaluate $f(x)$ at $x = h$ to find k.

We often want to find the roots, or *x*-intercepts, of a quadratic function. There are a few methods to determine the roots, each of which is useful for various problems:

- When the function is easily factorable, factor it, set the factors equal to zero, and solve for x.
- When the quadratic function is in general form, use the quadratic formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
- Rewrite the quadratic function in standard form, set it equal to zero, and solve for x. (If you know the method of completing the square, this is very similar.

Example Problems

1. Write the following quadratic function in standard form: $f(x) = 2x^2 + 4x + 1$

$$h = -\frac{b}{2a} = -\frac{4}{2*2} = -\frac{4}{4} = -1$$
$$k = f(-1) = 2(-1)^2 + 4(-1) + 1 = 2(1) - 4 + 1 = -1$$
$$f(x) = a(x-h)^2 + k$$
$$f(x) = 2\left(x - (-1)\right)^2 - 1$$
$$f(x) = 2(x+1)^2 - 1$$

Reminders

Thank you for reading this resource! Here are a few reminders about other Precalculus opportunities:

- Group Tutoring for Precalculus is on Tuesdays from 5:00 to 6:00 p.m. through Microsoft Teams. If you have not received an email inviting you to sign up, and you want to attend, please email me at Sydney_Schirner1@baylor.edu, and I will make sure you are included!
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/support_programs/index.php?id=955694.
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at https://www.baylor.edu/tutoring.

References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: https://openstax.org/details/books/precalculus