Modeling Dust-Density Wave Fields as a System of Coupled van der Pol Oscillators

Kristoffer Ole Menzel, Tim Bockwoldt, Oliver Arp, Alexander Piel

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Dust density wave fields

- Dust-density waves (DDWs) are attributed to a Buneman-type instability
- Energy gain (by streaming ions) competes with damping (neutral gas)
- Extended 3D wave fields can be observed under µg conditions
- DDWs exhibit complicated spatio-temporal dynamics and have a highly nonlinear character
Experimental setup

- Symmetric parallel plate rf discharge (13.56 MHz)
- Gas pressure: 5-35 Pa (Ar)
- Rf voltage: 40-70 $V_{pp}$
- CMOS camera (100fps)

Monodisperse MF particles (diameter: 6.8µm und 9.55µm)
Frequency distribution

- Frequency varies spatially
- Generation of frequency clusters
- Defects occur at the cluster boundaries

Huygens
Frequency distribution

Results suggest a new interpretation of DDWs!

- Frequency varies spatially
- Huygens
- Generation of frequency clusters
- Defects occur at the cluster boundaries
Numerical studies - model

Chain of coupled, self-excited, nonlinear oscillators with frequency gradient

\[ \ddot{x}_i - \epsilon (1 - \beta x_i^2) \omega_{0i} \dot{x}_i + \omega_{0i}^2 x_i = g [\dot{x}_{i-1} + \dot{x}_{i+1} - 2 \dot{x}_i] \]

- van der Pol oscillator
- nonlinearity
- amplitude saturation
- frequency gradient \( \omega_{0i} = \omega_{00} + i \cdot \Delta \)
- coupling
Numerical studies – frequency distribution

Clusters with incommensurable frequencies occur.
Numerical studies – frequency distribution

Clusters with incommensurable frequencies occur

Wave propagation in the direction of lower frequencies
Numerical studies – frequency distribution

**frequency distribution**

Clusters with incommensurable frequencies occur

**space-time diagram**

Wave propagation in the direction of lower frequencies

Defects occur exclusively at the cluster boundaries
Numerical studies – parameter variation

**Coupling**

- Increasing coupling
- Increasing frequency gradient
- Increasing nonlinearity

**Nonlinearity**

- Cluster size decreases with:
  - Increasing coupling
  - Decreasing frequency gradient
  - Increasing nonlinearity
Comparison between model and experiment

\[ \omega_{pd} = \left( \frac{n_d Q_d^2}{\varepsilon_0 m_d} \right)^{1/2} \]

- **p=30 Pa, U_{rf}=65V_{pp}, d=6.8\mu m**
- **p=15 Pa, U_{rf}=70V_{pp}, d=6.8\mu m**
Comparison between model and experiment

\[ \omega_{pd} = \left( \frac{n_d Q_d^2}{\varepsilon_0 m_d} \right)^{1/2} \]

Model works fine for situations with incommensurable frequencies.
Clusters with commensurable frequencies occur at different plasma conditions.

Situation is similar to that in magnetized anodic plasmas with external modulation.

Numerical studies - global modulation

Chain of coupled, self-excited, nonlinear oscillators with frequency gradient

van der Pol oscillator

external forcing

\[ \ddot{x}_i - \epsilon (1 - \beta x_i^2) \omega_0 i \dot{x}_i + \omega_0^2 x_i = g [\dot{x}_{i-1} + \dot{x}_{i+1} - 2\dot{x}_i] \]

\[ \ddot{x}_i - \epsilon (1 - \beta x_i^2) \omega_0 i \dot{x}_i + \omega_0^2 x_i = g [\dot{x}_{i-1} + \dot{x}_{i+1} - 2\dot{x}_i] + F \cdot \sin(\omega t) \]
Numerical studies - global modulation

Chain of coupled, self-excited, nonlinear oscillators with frequency gradient

van der Pol oscillator

\[ \ddot{x}_i - \epsilon (1 - \beta x_i^2) \omega_{0i} \dot{x}_i + \omega_{0i}^2 x_i = g[\dot{x}_{i-1} + \dot{x}_{i+1} - 2\dot{x}_i] \]

external forcing

\[ \ddot{x}_i - \epsilon (1 - \beta x_i^2) \omega_{0i} \dot{x}_i + \omega_{0i}^2 x_i = g[\dot{x}_{i-1} + \dot{x}_{i+1} - 2\dot{x}_i] + F \cdot \sin(\omega t) \]

parametric excitation

\[ \ddot{x}_i - \epsilon [1 - \beta (1 + F \cdot \sin(\omega t)) x_i^2] \omega_{0i} \dot{x}_i + \omega_{0i}^2 x_i = g[\dot{x}_{i-1} + \dot{x}_{i+1} - 2\dot{x}_i] \]
Numerical studies - global modulation

\[ \omega_{ext} = 1.027 \]

- **External Forcing**
  - Harmonic synchronization

- **Parametric Excitation**
  - Superharmonic synchronization
Experiment - global modulation

Plasma glow and electrode voltage are modulated significantly

- DDWs are modulated globally by internal sources
- DDWs can be treated as a self-organized system containing the dust, the plasma and the external circuit
Summary

• Dust density wave fields exhibit a complicated spatio-temporal behavior, which is reflected in the occurrence of frequency clusters.

• The experimental observations lead to the conclusion that the DDWs can be modeled as an ensemble of van der Pol oscillators.

• The frequency distributions show clusters of incommensurable or commensurable values depending on the system parameters.

• The latter case can be treated in the model by including an additional external and/or parametric excitation.

Thank you for the attention!

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3rd International Workshop on Diagnostics and Simulation of Dusty Plasmas
Kiel, 26-28 August 2012
www.physik.uni-kiel.de/dsdp

Tutorial Lectures

Oliver Arp (Kiel)
Dusty plasmas under microgravity

Franz-Xaver Bronild (Greifswald)
Charging of dust grains

Franko Greiner (Kiel)
Nanodust in magnetized plasmas

Ian Hutchinsen (MIT, Cambridge)
Dust particle wakes in flowing plasmas

André Melzer (Greifswald)
Diagnostics of dusty plasmas

Eva Kovacevic (Orleans)
Growth of dust particles

Jiri Pavlu (Prag)
Dust particles in traps

Zoltan Sternovsky (Boulder)
Lunar dust

The workshop is intended to gather students, young researchers and experts in the field of dusty plasmas. It comprises of tutorial lectures, as well as oral presentations and posters reporting recent progress in this field.

Venue: CAU, Physics Centre, Leibnizstr. 13, Kiel, Germany
Loc: PD Dr. Dietmar Block
Email: dsdp@physik.uni-kiel.de

Christian-Albrechts-University, Kiel
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"Fundamentals of Complex Plasmas"