On the convergence of Hermite–Padé approximants.

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Markov's functions

Let s be a finite Borel measure with constant sign whose compact support consists of infinitely many points and is contained in the real line. By Δ we denote the smallest interval which contains the support of s. We denote this class of measures by $\mathcal{M}(\Delta)$. Let

$$\hat{s}(z) = \int_{\Delta} \frac{ds(x)}{z-x}$$

denote the Cauchy transform of s.

Padé approximants

Definition (Padé approximants)

Fix a non zero number $n \in \mathbb{N}$, then there exists a polynomials P_n and Q_n such that

i) $deg\ P_n \leq n-1$, $deg\ Q_n \leq n \quad Q_n \not\equiv 0$

ii) $Q_n(z)\hat{s}(z) - P_n(z) = \mathcal{O}(1/z^{n+1}), \qquad z \to \infty,$

The unique rational function $\frac{P_n}{Q_n}$ is called diagonal Padé approximants of $\hat{s}$.

Markov's theorem

Theorem (A.A. Markov,1895)

If Δ is bounded, we have

$$\lim_{n\to\infty} \frac{P_n(z)}{Q_n(z)} = \widehat{s}(z), \quad \mathcal{K} \subset \overline{\mathbb{C}} \setminus \Delta.$$

Markov, A. A. Deux demonstrations de la convergence de certains fractions continues. *Acta Math.* 19 (1895), 93–104.

- Q_n is the n-orthogonal polynomial respect to the measure s
- Q_n has n simples zeros on Δ

Riemann-Hilbert problem for Nikishin systems

- RH-1 $H(z) : \mathbb{C} \setminus \left(\cup_{j=1}^{m} \Delta_j\right) \to \mathbb{C}^{(m+1)\times(m+1)}$ is analytic;
- RH-2 $H^{+}(x) = S(x) \cdot H^{-}(x)$ for $x \in \cup_{j=1}^{m} \Delta_j$;
- RH-3

$$H(z) = \left(I + \mathcal{O}\left(\frac{1}{z}\right)\right)$$

as $z \to \infty$.

$$S(x) = \begin{pmatrix} 1 & s_1(x) & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & s_2(x) & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & s_3(x) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & s_m(x) \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

Nikishin system

$$\begin{pmatrix} 1 & \widehat{s}_{1,1} & \widehat{s}_{1,2} & \widehat{s}_{1,3} & \dots & \widehat{s}_{1,m-1} & \widehat{s}_{1,m} \\ 0 & 1 & \widehat{s}_{2,2} & \widehat{s}_{2,3} & \dots & \widehat{s}_{2,m-1} & \widehat{s}_{2,m} \\ 0 & 0 & 1 & \widehat{s}_{3,3} & \dots & \widehat{s}_{3,m-1} & \widehat{s}_{3,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & \widehat{s}_{m,m} \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

We say that $\widehat{\mathbf{s}} := (1, \widehat{s}_{1,1}, \dots, \widehat{s}_{1,m})$ is the Nikishin system of functions generating by the measures $\mathcal{N}(s_1, \dots, s_m)$

$(1, \widehat{s}_{j,j}, \dots, \widehat{s}_{j,m})$ is the Nikishin system of functions generating by the measures $\mathcal{N}(s_j, \dots, s_m)$

Nikishin system

$$\begin{pmatrix} 1 & \widehat{s}_{1,1} & \widehat{s}_{1,2} & \widehat{s}_{1,3} & \dots & \widehat{s}_{1,m-1} & \widehat{s}_{1,m} \\ 0 & 1 & \widehat{s}_{2,2} & \widehat{s}_{2,3} & \dots & \widehat{s}_{2,m-1} & \widehat{s}_{2,m} \\ 0 & 0 & 1 & \widehat{s}_{3,3} & \dots & \widehat{s}_{3,m-1} & \widehat{s}_{3,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & \widehat{s}_{m,m} \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

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$(1, \widehat{s}_{j,j}, \dots, \widehat{s}_{j,m})$ is the Nikishin system of functions generating by the measures $\mathcal{N}(s_j, \dots, s_m)$

Type II Hermite-Padé approximants

Definition (Type II Hermite-Padé approximants)

Let $\widehat{\mathbf{s}} := (1, \widehat{s}_{1,1}, \ldots, \widehat{s}_{1,m})$ *be a Nikishin system of functions. Fix a non zero multi-index* $\vec{n} = (n_1, \ldots, n_m) \in \mathbb{N}_+^m$*,* $|\vec{n}| = n_1 + \ldots + n_m$*. There exists a polynomial* $Q_{\vec{n}}$ *such that*

i) $deg\ Q_{\vec{n}} \leq |\vec{n}| \quad Q_{\vec{n}} \not\equiv 0$

ii) $Q_{\vec{n}}(z)\hat{s}_{1,j}(z) - P_{\vec{n},j}(z) = \mathcal{O}(1/z^{n_j+1}), \qquad z \to \infty, \qquad j = 1, \ldots, m$

for some polynomials $(P_{\vec{n},1}, \ldots P_{\vec{n},m})$*. The vector of rational functions* $(\frac{P_{\vec{n},1}}{Q_{\vec{n}}}, \ldots, \frac{P_{\vec{n},m}}{Q_{\vec{n}}})$ *is called type II Hermite-Padé approximants of* $\hat{s}$ *respect to the multi-índex* $\vec{n}$*.*

If $m = 1$, $\frac{P_{\vec{n},1}}{Q_{\vec{n}}}$ is the classical Padé approximant of $\hat{s}_{1,1}$.

Analog of Markov's Theorem for type II

Theorem

For $j = 1, \ldots, m$

$$\lim_{|\vec{n}| \to \infty} \frac{P_{\vec{n},j}(z)}{Q_{\vec{n}}(z)} = \widehat{s}_{1,j}(z), \quad \mathcal{K} \subset \overline{\mathbb{C}} \setminus \Delta_1.$$

J. Bustamante and G. López Lagomasino Hermite–Padé approximation for Nikishin systems of analytic functions. *Mat. Sb.* 183 (1992), 117–138 (Russian); English translation in Russian Acad. Sci. Sb. Math. 77 (1994), 367–384.

Type I Hermite-Padé approximants

Definition (Type I Hermite-Padé approximants)

Let $\widehat{\mathbf{s}} := (1, \widehat{s}_{1,1}, \ldots, \widehat{s}_{1,m})$ *be a Nikishin system of functions. Fix a non zero multi-index* $\vec{n} = (n_0, \ldots, n_m) \in \mathbb{N}_+^{m+1}$*, where* $n_0 - 1 \geq n_j, j = 1, \ldots m$*. There exist polynomials* $a_{\vec{n},0}, a_{\vec{n},1}, \ldots, a_{\vec{n},m}$*, not all identically equal to zero, such that*

i) $\deg a_{\vec{n},j} \leq n_j - 1, j = 0, \ldots, m$

ii) $a_{\vec{n},0}(z) + \sum_{j=1}^{m} a_{\vec{n},j}(z)\hat{s}_{1,j}(z) = \mathcal{O}(1/z^{|\vec{n}|-n_0}), z \to \infty,$

The vector of polynomials $a_{\vec{n},0}, a_{\vec{n},1}, \ldots, a_{\vec{n},m}$ *is called type I Hermite-Padé polynomials of* $\hat{s}$ *respect to the multi-índex* $\vec{n}$*.*

If $m = 1$, $\frac{a_{\vec{n},0}}{a_{\vec{n},1}}$ is the classical Padé approximants of $\hat{s}_{1,1}$.

Analog of Markov's Theorem for type I

Theorem

For $j = 0, \ldots, m-1$

$$\lim_{|\vec{n}|\to\infty} \frac{a_{\vec{n},j}}{a_{\vec{n},m}}(z) = (-1)^{m-j}\hat{s}_{m,j+1}(z) \quad \mathcal{K} \subset \overline{\mathbb{C}} \setminus \Delta_m.$$

G. López Lagomasino and S. Medina Peralta. On the convergence of type I Hermite-Padé approximants. *Advances in Math.* **337** *(2015), 1077-1141..*

Dual Nikishin systems

- RH-1 $H(z) : \mathbb{C} \setminus \left(\cup_{j=1}^{m} \Delta_j\right) \to \mathbb{C}^{(m+1)\times(m+1)}$ is analytic;
- RH-2 $H^{+}(x) = S(x) \cdot H^{-}(x)$ for $x \in \cup_{j=1}^{m} \Delta_j$;
- RH-3
$$H(z) = \left(I + \mathcal{O}\left(\frac{1}{z}\right)\right)$$
as $z \to \infty$.

$$S(x) = \begin{pmatrix} 1 & s_m(x) & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & s_{m-1}(x) & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & s_{m-2}(x) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & s_1(x) \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

Nikishin system

$$\begin{pmatrix} 1 & \widehat{s}_{m,m} & \widehat{s}_{m,m-1} & \widehat{s}_{m,m-2} & \cdots & \widehat{s}_{m,2} & \widehat{s}_{m,1} \\ 0 & 1 & \widehat{s}_{m-1,m-1} & \widehat{s}_{m-1,m-2} & \cdots & \widehat{s}_{m-1,2} & \widehat{s}_{m-1,1} \\ 0 & 0 & 1 & \widehat{s}_{m-2,m-2} & \cdots & \widehat{s}_{m-2,2} & \widehat{s}_{m-2,1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 1 & \widehat{s}_{1,1} \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \end{pmatrix}$$

Extensions of Markov's theorem

Uniform Convergence	$\widehat{s}$		
Padé approximants	✓		

Extensions of Markov's theorem

Uniform Convergence	$\widehat{s}$	$\widehat{s}+r$ (Real case)	$\widehat{s}+r$ (Complex case)
Padé approximants	✓		

Extensions of Markov's theorem

Uniform Convergence	$\widehat{s}$	$\widehat{s}+r$ (Real case)	$\widehat{s}+r$ (Complex case)
Padé approximants	✓	✓	✓

A.A. Gonchar. On the convergence of Padé approximants for some classes of meromorphic functions. *Mat. Sb.* **97(139)** (1975), 607-629.; English transl. in Math. USRR sb 26(1975).

E.A. Rakhmanov. On the convergence of diagonal Padé approximants. *Matem Sb.* **104** (1977), 271–291 (Russian); English transl. in Math. USSR Sb. **33** (1977), 243–260.

Extensions of Markov's theorem

Let $r := (r_1, \ldots, r_m)$ be a system of real rational functions where the poles of r_j lie in $\mathbb{C} \setminus (\Delta_1 \cup \Delta_m)$ and consider the system of functions $(1, \widehat{s}_{1,1} + r_1, \ldots, \widehat{s}_{1,m} + r_m)$.

Uniform Convergence	Nikishin system	Real case	Complex case
Type II Hermite-Padé approximants	✓		
Type I Hermite-Padé approximants	✓		

Extensions of Markov's theorem

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Uniform Convergence	Nikishin system	Real case	Complex case
Type II Hermite-Padé approximants	✓		
Type I Hermite-Padé approximants	✓	✓	

G. López Lagomasino and S. Medina Peralta. On the convergence of type I Hermite-Pade approximants for rational perturbations of a Nikishin system. *Journal of Computational and Applied Mathematics (2015), 284, 216-227.*

Extensions of Markov's theorem

Let $r := (r_1, \dots, r_m)$ be a system of real rational functions where the poles of r_j lie in $\mathbb{C} \setminus (\Delta_1 \cup \Delta_m)$ and consider the system of functions $(1, \widehat{s}_{1,1} + r_1, \dots, \widehat{s}_{1,m} + r_m)$.

Uniform Convergence	Nikishin system	Real case	Complex case
Type II Hermite-Padé approximants	✓	✓	
Type I Hermite-Padé approximants	✓	✓	

U. Fidalgo Prieto, G. López Lagomasino and S. Medina Peralta.
Hermite-Padé approximation for certain systems of meromorphic functions.
Matematicheskii Sbornik (2015) 206, 57-76.

Where are the zeros of the multiple orthogonal polynomials?

Real case

Algebraic properties.
$(t_0, t_1\hat{s}_{1,1}, \ldots, t_m\hat{s}_{1,m})$ is an AT-system in Δ_1

Definition (Type I Hermite-Padé approximants)

Let $\widehat{\mathbf{s}} := (1, \widehat{s}_{1,1}, \ldots, \widehat{s}_{1,m})$ *be a Nikishin system of functions. Fix a non zero multi-index* $\vec{n} = (n_0, \ldots, n_m) \in \mathbb{N}_+^{m+1}$*, where* $n_0 - 1 \geq n_j + d_j, j = 1, \ldots m$*. There exist polynomials* $p_{\vec{n},0}, p_{\vec{n},1}, \ldots, p_{\vec{n},m}$*, not all identically equal to zero, such that*

i) $\deg p_{\vec{n},j} \leq n_j - 1, j = 0, \ldots, m,$

ii) $p_{\vec{n},0}(z) + \sum_{j=1}^{m} p_{\vec{n},j}(z)t_j(z)\widehat{s}_{1,j}(z) = \mathcal{O}(1/z^{|\vec{n}-n_0|}) \in \mathcal{H}(\mathbb{C} \setminus \Delta_1).$

Analog of Markov's Theorem for type I

Theorem

For $j = 0, \ldots, m-1$

$$\lim_{|\vec{n}|\to\infty} \frac{p_{\vec{n},j}}{p_{\vec{n},m}}(z) = (-1)^{m-j}\frac{t_m}{t_j}\widehat{s}_{m,j+1}(z) \quad \mathcal{K} \subset (\mathbb{C} \setminus \Delta_m)'$$

U. Fidalgo Prieto, G. López Lagomasino and S. Medina Peralta.
Hermite-Padé approximation for certain systems of meromorphic functions.
Matematicheskii Sbornik (2015) 206, 57-76.

Complex case

Theorem

For $j = 0, \dots, m-1$

$$\lim_{|\vec{n}|\to\infty} \frac{p_{\vec{n},j}}{p_{\vec{n},m}}(z) = (-1)^{m-j}\frac{t_m}{t_j}\widehat{s}_{m,j+1}(z) \quad \mathcal{K} \subset (\mathbb{C} \setminus \Delta_m)'$$

- Asymptotic behavior. The measure have to satisfies some conditions ($s_j' > 0$ a.e. on Δ_j)
- Decreasing multi-indices, $n_j > n_{j+1} + d_{j+1}, j = 0, \dots m-1.$
- The zeros of t_j lie on $\mathbb{C} \setminus \left(\cup_{j=1}^{m} \Delta_j\right)$

Relative asymptotic

Theorem

For $j = 0, \ldots, m$

$$\lim_{|\vec{n}| \to \infty} \frac{p_{\vec{n},j} t_j(z)}{a_{\vec{n},j}(z)} = \mathcal{F}(z; (t_1, \ldots, t_m)), \qquad \textit{inside} \qquad (\mathbb{C} \setminus \Delta_m),$$

where the function $\mathcal{F}(z; (t_1, \ldots, t_m))$ *is analytic in* $\mathbb{C} \setminus \Delta_m$ *and have a zero of order* τ *at the point* ζ *if* ζ *is a zero of order* τ *for some polynomial* t_j*,* $j = 1, \ldots, m$.

- There exists $N > 0$, such that for all $\vec{n}$ with $|\vec{n}| \geq N$ the multi-indices $\vec{n}$ are normal with respect to the system of functions $(1, t_1 \widehat{s}_{1,1}, \ldots, t_m \widehat{s}_{1,m})$.
- if ζ is a zero of some t_k of order κ then for each $j = 1, \ldots, m$, $j \neq k$ ζ is an κ attractor of the zeros of $p_{\vec{n},j}$ and the remaining zeros of $p_{\vec{n},j}$ accumulate on Δ_m.

- $\mathcal{A}_{\vec{n},0} := a_{\vec{n},0}(z) + a_{\vec{n},1}(z)\hat{s}_{1,1}(z) + a_{\vec{n},2}(z)\hat{s}_{1,2}(z) = \mathcal{O}(1/z^{n_1+n_2})$
- $\mathcal{A}_{\vec{n},1}(x) := a_{\vec{n},1}(x) + a_{\vec{n},2}(x)\widehat{s}_{2,2}(x)$
- $\int x^\nu \mathcal{A}_{\vec{n},1}(x) ds_{1,1}(x) = 0 \quad \nu = 0, \ldots, n_1 + n_2 - 2$

- $\mathcal{P}_{\vec{n},0} := p_{\vec{n},0}(z) + p_{\vec{n},1}(z)t_1(z)\hat{s}_{1,1}(z) + p_{\vec{n},2}(z)t_2(z)\hat{s}_{1,2}(z) = \mathcal{O}(1/z^{n_1+n_2})$
- $\mathcal{P}_{\vec{n},1}(x) := p_{\vec{n},1}(x)t_1(x) + p_{\vec{n},2}(x)t_2(x)\widehat{s}_{2,2}(x)$
- $\int x^\nu \mathcal{P}_{\vec{n},1}(x) ds_{1,1}(x) = 0 \quad \nu = 0, \ldots, n_1 + n_2 - 2$

Ratio asymptotic

Consider the (3)-sheeted Riemann surface

$$\mathcal{R} = \overline{\bigcup_{j=0}^{2} \mathcal{R}_j},$$

formed by the consecutively "glued" sheets

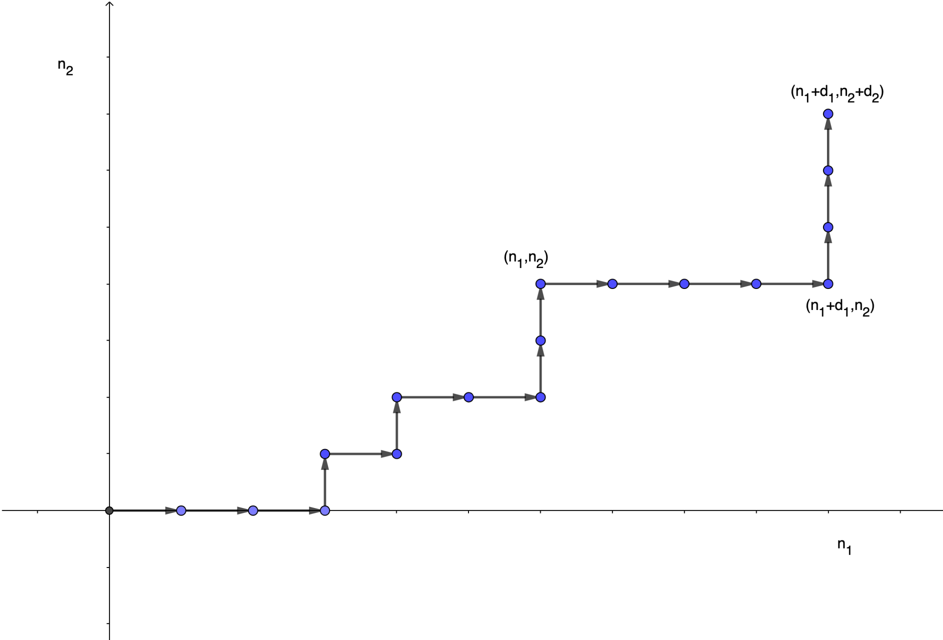
$$\mathcal{R}_0 := \overline{\mathbb{C}} \setminus \Delta_1, \quad \mathcal{R}_1 := \overline{\mathbb{C}} \setminus \{\Delta_1 \cup \Delta_2\}, \qquad \mathcal{R}_2 = \overline{\mathbb{C}} \setminus \Delta_2,$$

where the upper and lower banks of the slits of two neighboring

Ratio asymptotic

Let $\psi^{(1)} : \mathcal{R} \longrightarrow \overline{\mathbb{C}}$ denote a conformal mapping whose divisor consists of one simple zero at the point $\infty^{(0)} \in \mathcal{R}_0$ and one simple pole at the point $\infty^{(1)} \in \mathcal{R}_1$. Let $\psi^{(2)} : \mathcal{R} \longrightarrow \overline{\mathbb{C}}$ denote a conformal mapping whose divisor consists of one simple zero at the point $\infty^{(0)} \in \mathcal{R}_0$ and one simple pole at the point $\infty^{(2)} \in \mathcal{R}_2$.

- $\lim_{|\vec{n}|\to\infty} \frac{a_{\vec{n},2}}{a_{\vec{n}+e_1,2}}(z) = \frac{1}{\psi^{(1)}}(z) \qquad \lim_{|\vec{n}|\to\infty} \frac{a_{\vec{n},2}}{a_{\vec{n}+e_2,2}}(z) = \frac{1}{\psi^{(2)}}(z) \quad z \in \mathcal{R}_2$
- $A\left(\frac{1}{\psi^{(2)}}\right) + B = \frac{1}{\psi^{(1)}}$


n2
(n1+d1,n2+d2)
(n1,n2)
(n1+d1,n2)
n1

- $\mathcal{P}_{\vec{n}+\vec{d},1} = \sum_{k=1}^{|\vec{n}+\vec{d}|} \lambda_{\overrightarrow{n_k}} \mathcal{A}_{\overrightarrow{n_k},1}$
- Using the orthogonality relations $\mathcal{P}_{\vec{n}+\vec{d},1} = \sum_{k=|\vec{n}|}^{|\vec{n}+\vec{d}|} \lambda_{\overrightarrow{n_k}} \mathcal{A}_{\overrightarrow{n_k},1}$
- Since Nikishin system are perfect $t_j p_{\vec{n}+\vec{d},j} = \sum_{k=|\vec{n}|}^{|\vec{n}+\vec{d}|} \lambda_{\overrightarrow{n_k}} a_{\overrightarrow{n_k},j}$
- $\frac{t_2 p_{\vec{n}+\vec{d},2}}{\lambda_{\vec{n}} a_{\vec{n}+\vec{d},2}} = \sum_{k=|\vec{n}|}^{|\vec{n}+\vec{d}|} \lambda_{\overrightarrow{n_k}} \frac{a_{\overrightarrow{n_k},2}}{\lambda_{\vec{n}} a_{\vec{n}+\vec{d},2}}$
- $\lim_{|\vec{n}|\to\infty \ \vec{n}\in\Lambda_1} \frac{t_2 p_{\vec{n}+\vec{d},2}}{\lambda_{\vec{n}} a_{\vec{n}+\vec{d},2}} = Pol_{\Lambda_1}\left(\frac{1}{\psi^{(2)}}\right)$ with at most degree $d_1 + d_2$.

$$\lim_{\vec{n}\in\Lambda_1} \frac{t_1 p_{\vec{n},1}}{t_2 p_{\vec{n},2}} = \widehat{s}_{2,2}$$

$$\lim_{\vec{n}\in\Lambda_1} \frac{t_1 p_{\vec{n},1} - \widehat{s}_{2,2} t_2 p_{\vec{n},2}}{t_2 p_{\vec{n},2}}$$

$$= \lim_{\vec{n}\in\Lambda_1} \frac{\sum_{k=0}^{D} \lambda_{\vec{n},k} \left(a_{\vec{n}_k,1} - \widehat{s}_{2,2} a_{\vec{n}_k,2}\right)}{\lambda_{\vec{n}} p_{\vec{n},2} t_2}$$

$$= \lim_{\vec{n}\in\Lambda_1} \frac{\sum_{k=0}^{D} \lambda_{\vec{n},k} \left(a_{\vec{n}_k,1} - \widehat{s}_{2,2} a_{\vec{n}_k,2}\right)}{a_{\vec{n}_D,2}} \frac{a_{\vec{n}_D,2}}{\lambda_{\vec{n}} p_{\vec{n},2} t_2}$$

$$= \lim_{\vec{n}\in\Lambda_1} \left(\sum_{k=0}^{D} \lambda_{\vec{n},k} \left(\frac{a_{\vec{n}_k,1} - \widehat{s}_{2,2} a_{\vec{n}_k,2}}{a_{\vec{n}_k,2}} \right) \frac{a_{\vec{n}_k,2}}{a_{\vec{n}_D,2}} \right) \frac{a_{\vec{n}_D,2}}{\lambda_{\vec{n}} p_{\vec{n},2} t_2} = 0.$$

- if ζ is a zero of t_1, or t_2 of order κ then ζ is an κ attractor of the zeros of $p_{\vec{n},1}$ and $p_{\vec{n},2}$ and the remaining zeros of $p_{\vec{n},1}$ and $p_{\vec{n},2}$ accumulate on Δ_2.
- $\lim_{|\vec{n}|\to\infty} \frac{t_2 p_{\vec{n}+\vec{d},2}}{\lambda_{\vec{n}} a_{\vec{n}+\vec{d},2}} = Pol\left(\frac{1}{\psi^{(2)}}\right)$. The limt does not depend on the subsequence Λ_1.

THANK YOU!!!