Please do not write in these boxes.

1	2	3	4	5	6	7	8	9	10	11	12	13	TOTAL

MTH 1321
Spring 2021 Departmental Final Exam

Name:________________________________

Instructor:________________________________

NO CALCULATOR ALLOWED

Complete the following in the space provided. Show the steps or reasoning leading to your answer for full credit.

1. Complete this table of derivatives:

$(\sin x)' =$ $\qquad$ $(\csc x)' =$

$(\cos x)' =$ $\qquad$ $(\sec x)' =$

$(\tan x)' =$ $\qquad$ $(\cot x)' =$

$(a^x)' =$ $\qquad$ $(\log_a x)' =$

$(\sin^{-1} x)' =$ $\qquad$ $(\tan^{-1} x)' =$

2. The table below summarizes recent global CO_2 emissions from fossil fuel energy sources, measured in gigatons (1 gigaton = 1 billion metric tons) at time t years since 2010.

t (years since 2010)	0	2	4	6	8
CO_2 (gigatons)	33	68	104	139	177

Source: `www.globalcarbonproject.org`

Suppose that global CO_2 emissions is a differentiable function for all time.

(a) What does the Intermediate Value Theorem say must be true about global CO_2 emissions from 2010 to 2018?

(b) What does the Mean Value Theorem say must be true about global CO_2 emissions from 2010 to 2018?

3. Evaluate the following limits.

   (a) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

   (b) $\lim_{x \to \infty} \frac{xe^x + 2x - 1}{3 + x}$

4. Compute the following. You do not need to simplify your answer.

   (a) Let $f(x) = \cos(x^3) \ln x$. Find $f'(x)$.

   (b) Let $y = \frac{e^x}{1 - x^2}$. Find $\frac{dy}{dx}$.

5. Compute the following:

(a) $\int (\sin x + \cos x + \sqrt{x} + 10^x)\, dx$

(b) $\int e^{3x+1}\, dx$

6. Compute the following:

(a) $\int_0^{\pi/3} \sec^2 x\, dx$

(b) $\int_0^1 \frac{x^3}{x^4+9}\, dx$

7. Let $f(x) = \dfrac{1}{\sqrt{x}}$.

   (a) Find the linear approximation $\ell(x)$ (i.e., the equation of the tangent line) to f at $x = 1$.

   (b) Use $\ell(x)$ to approximate $\dfrac{1}{\sqrt{1.2}}$.

8. Answer the following.

   (a) For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?
   (A) The change in temperature during the first day.
   (B) The change in temperature during the 24th hour.
   (C) The average rate at which the temperature changed during the 24th hour.
   (D) The instantaneous rate at which the temperature is changing during the first day.
   (E) The instantaneous rate at which the temperature is changing at the end of the 24th hour.

   (b) Which Riemann sum equals $\displaystyle\int_0^3 x^2\,dx$ if we use the right-endpoint method with N subintervals of equal length?

   (A) $\displaystyle\sum_{i=1}^{N} \left(\frac{3i}{N}\right)^2 \cdot \frac{3}{N}$

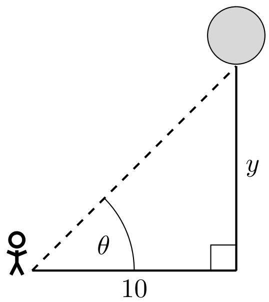
   (B) $\displaystyle\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{3i}{N}\right)^2 \cdot \frac{3}{N}$

   (C) $\displaystyle\sum_{i=1}^{N} \left(\frac{3i}{N}\right)^2$

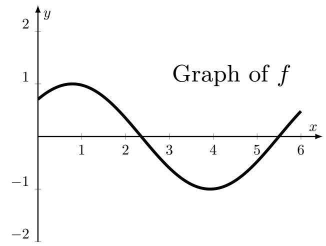
   (D) $\displaystyle\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{3i}{N}\right)^2$

   (E) None of the above

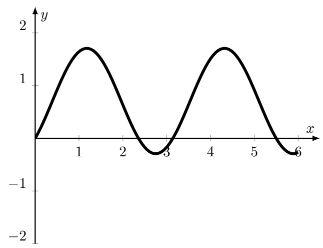
9. A hot air balloon rising straight up from a level field is tracked by a person on the ground 10 ft away from the point of lift-off. Let θ denote the angle between the ground and the person's line of sight as shown. At the moment that $\theta = \pi/4$ radians, θ is increasing at a rate of 0.7 radians per minute. How fast is the balloon rising at that moment? Include units.

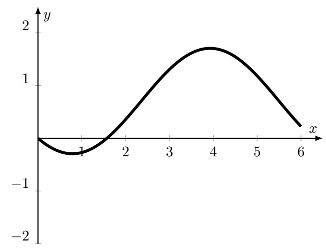


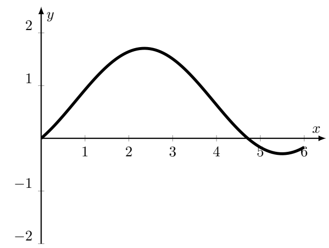
10. (a) The graph of $y = f(t)$ is shown below.

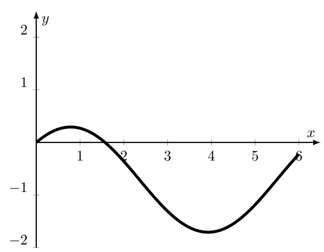


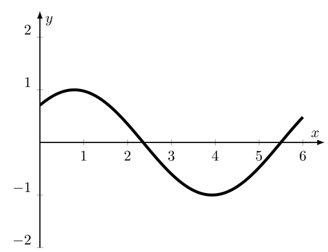
Let $F(x) = \int_0^x f(t)\,dt$. Which of the following best matches the graph of $y = F(x)$? (All graphs are on the same scale.)

(A) 

(B) 

(C) 

(D) 

(E) 

(b) Find the absolute maximum and minimum values of $g(x) = x^3 - 3x$ on $[0, 4]$.

11. Your calculus professor walks back and forth in a straight line in front of the room while lecturing. Suppose the professor's position s (measured in feet, from the left edge of the board) at time t (in minutes) is given by $s(t) = t - \ln(2t + 1)$ for $t \geq 0$.

(a) Find your professor's velocity and acceleration when $t = 1$. Include units.

(b) Set up—but do not evaluate—an integral for the total distance that your professor travels from $t = 0$ to $t = 3$.

12. Mark each of the following statements as true (T) or false (F). You do *not* need to justify your answer.

(a) If f and g are differentiable for all real numbers, then $(f + g)' = f' + g'$ and $(fg)' = f'g'$.

(b) $\displaystyle\lim_{h \to 0} \frac{\sin(\pi + h) - \sin \pi}{h} = 1$

(c) If f is continuous on $[a, b]$, then $\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$.

(d) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

(e) If $g(x) = \int_2^x \ln t\,dt$, then $g'(2) = 0$.

13. Let $f(x) = xe^{-x^2}$.

(a) Find the x-coordinate of all critical points of f. Determine whether a local max, local min, or neither occurs there. Justify your answer.

(b) Find the exact area of the shaded region shown.