Complete the following in the space provided. Show the steps or reasoning leading to your answer for full credit.

1. Let \( f(x) = \begin{cases} \sqrt{x}, & 0 \leq x \leq 4, \\ mx, & x > 4, \end{cases} \) where \( m \) is a constant.

   (a) Is there any value of \( m \) which makes \( f \) continuous at \( x = 4 \)? If so, find it. If not, explain why not.

   (b) Find \( \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} \).

2. Let \( f(x) = x^3 \). Use the limit definition of the derivative to show that \( f'(x) = 3x^2 \).
3. Let \( f(x) = \frac{3x^2 + 8}{(2x + 1)^2} \).

(a) Find all horizontal asymptotes for the graph of \( y = f(x) \). Justify your answer using an appropriate limit.

(b) Find the slope of the line tangent to \( y = f(x) \) at \( x = 0 \).

4. A particle moves along the \( x \)-axis with velocity \( v(t) = 6(t - 1)^2 \). When \( t = 1 \), the particle is at position \( x = 2 \).

(a) Find the acceleration of the particle when \( t = 3 \).

(b) Find the position of the particle when \( t = 3 \).
5. Compute the following:

(a) Let \( f(x) = 2019 + \tan^{-1}(x) - \sqrt{x} \). Find \( f'(x) \).

(b) Let \( g(x) = e^{-x} \cos(2x) \). Find \( \frac{dg}{dx} \). You do not need to simplify your answer.

6. Compute the following:

(a) Let \( f(t) = \left[\ln(\sin t)\right]^2 \). Find \( f'(t) \). You do not need to simplify your answer.

(b) If \((x + 2y)y = 2x - y\), find \( \frac{dy}{dx} \) when \( x = 3 \) and \( y = 1 \).
7. Compute the following:

(a) \( \int (2019 + \sec x \tan x - e^x) \, dx \)

(b) \( \int_0^1 (\sqrt{x} - x) \, dx \)

8. Compute the following:

(a) \( \int_{-3}^2 \frac{2x}{x^2 + 5} \, dx \)

(b) \( \int \frac{x^2 + 5}{2x} \, dx \)
9. You begin to heat a pot of water on the stove. At time $t$ (in minutes), the temperature $T$ (in °F) of the water is recorded below. For $0 \leq t \leq 8$, $T$ is a differentiable function of $t$.

<table>
<thead>
<tr>
<th>$t$ (min)</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (°F)</td>
<td>100</td>
<td>110</td>
<td>140</td>
<td>160</td>
<td>180</td>
</tr>
</tbody>
</table>

(a) Find the average rate of change of the temperature of the water over $0 \leq t \leq 8$. Include units.

(b) Was there some time $t$ between $t = 0$ and $t = 8$ when the instantaneous rate of change of the temperature of the water was 10°F/min? Explain why or why not.

(c) Estimate $\int_{0}^{8} T(t) \, dt$ using a method of your choice.
10. A rectangle is inscribed in the first quadrant region bounded by the $x$-axis, the $y$-axis, and the parabola $y = 9 - x^2$ as shown below. That is, the base of the rectangle is along the $x$-axis, its lower left corner is at the origin, and its upper right corner is on the parabola $y = 9 - x$. Find the length and width of the rectangle of greatest perimeter.

11. The function $f$ is continuous for all values of $x$. Information about the sign of $f'$ and $f''$ is organized in the table below.

<table>
<thead>
<tr>
<th></th>
<th>$x &lt; 1$</th>
<th>$1 &lt; x &lt; 2$</th>
<th>$2 &lt; x &lt; 3$</th>
<th>$x &gt; 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of $f'$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>Sign of $f''$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Mark each of the following statements as true (T) or false (F). You do not need to justify your answer.

(a) $f$ has a local minimum at $x = 2$
(b) $f$ is decreasing and concave down at $x = 4$
(c) $f$ has an inflection point at $x = 1$
(d) $f'$ is decreasing at $x = 2.5$
(e) $f''$ has a local extremum at $x = 1$
12. Consider the function \( A(x) = \int_0^x \sin(t^2) \, dt \).

(a) Find \( A'(x) \).

(b) Find the \( x \)-value of the first positive critical number of \( A(x) \).

(c) At that first positive critical number you found in (b), does \( A(x) \) have a local max, a local min, or neither? Justify your answer.

13. You are bored so you begin to pour the salt out of a salt shaker at a constant rate of 3 cubic inches per second onto the table in such a way that it forms a conical pile whose height is always half the radius of the base. How fast is the base radius changing when the radius is 2 inches? Include units. (Recall that for a cone, \( V = \frac{1}{3} \pi r^2 h \).)