Please do not write in these boxes.

1	2	3	4	5	6	7	8	9	10	11	12	13	TOTAL

MTH 1321 Name:____________________________

Spring 2019 Departmental Final Exam Instructor:____________________________

NO CALCULATOR ALLOWED

Complete the following in the space provided. Show the steps or reasoning leading to your answer for full credit.

1. Let $f(x) = \begin{cases} \sqrt{x}, & 0 \leq x \leq 4, \\ mx, & x > 4, \end{cases}$ where m is a constant.

   (a) Is there any value of m which makes f continuous at $x = 4$? If so, find it. If not, explain why not.

   (b) Find $\lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1}$.

2. Let $f(x) = x^3$. Use the limit definition of the derivative to show that $f'(x) = 3x^2$.

3. Let $f(x) = \dfrac{3x^2 + 8}{(2x+1)^2}$.

(a) Find all horizontal asymptotes for the graph of $y = f(x)$. Justify your answer using an appropriate limit.

(b) Find the slope of the line tangent to $y = f(x)$ at $x = 0$.

4. A particle moves along the x-axis with velocity $v(t) = 6(t-1)^2$. When $t = 1$, the particle is at position $x = 2$.

(a) Find the acceleration of the particle when $t = 3$.

(b) Find the position of the particle when $t = 3$.

5. Compute the following:

(a) Let $f(x) = 2019 + \tan^{-1}(x) - \sqrt{x}$. Find $f'(x)$.

(b) Let $g(x) = e^{-x}\cos(2x)$. Find $\dfrac{dg}{dx}$. You do not need to simplify your answer.

6. Compute the following:

(a) Let $f(t) = [\ln(\sin t)]^2$. Find $f'(t)$. You do not need to simplify your answer.

(b) If $(x + 2y)y = 2x - y$, find $\dfrac{dy}{dx}$ when $x = 3$ and $y = 1$.

7. Compute the following:

(a) $\int (2019 + \sec x \tan x - e^x)\, dx$

(b) $\int_0^1 (\sqrt[3]{x} - x)\, dx$

8. Compute the following:

(a) $\int_{-3}^{2} \frac{2x}{x^2+5}\, dx$

(b) $\int \frac{x^2+5}{2x}\, dx$

9. You begin to heat a pot of water on the stove. At time t (in minutes), the temperature T (in °F) of the water is recorded below. For $0 \leq t \leq 8$, T is a differentiable function of t.

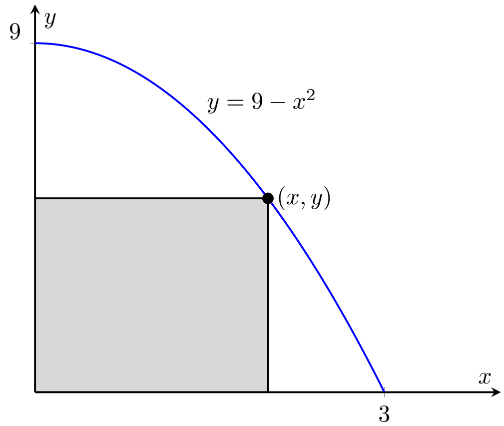
t (min)	0	1	3	4	8
T (°F)	100	110	140	160	180

(a) Find the average rate of change of the temperature of the water over $0 \leq t \leq 8$. Include units.

(b) Was there some time t between $t = 0$ and $t = 8$ when the instantaneous rate of change of the temperature of the water was 10°F/min? Explain why or why not.

(c) Estimate $\int_0^8 T(t)\, dt$ using a method of your choice.

10. A rectangle is inscribed in the first quadrant region bounded by the x-axis, the y-axis, and the parabola $y = 9 - x^2$ as shown below. That is, the base of the rectangle is along the x-axis, its lower left corner is at the origin, and its upper right corner is on the parabola $y = 9 - x^2$. Find the length and width of the rectangle of greatest perimeter.



11. The function f is continuous for all values of x. Information about the sign of f' and f'' is organized in the table below.

	$x < 1$	$1 < x < 2$	$2 < x < 3$	$x > 3$
Sign of f'	$-$	$-$	$+$	$-$
Sign of f''	$+$	$-$	$-$	$+$

Mark each of the following statements as true (T) or false (F). You do *not* need to justify your answer.

(a) f has a local minimum at $x = 2$

(b) f is decreasing and concave down at $x = 4$

(c) f has an inflection point at $x = 1$

(d) f' is decreasing at $x = 2.5$

(e) f' has a local extremum at $x = 1$

12. Consider the function $A(x) = \int_0^x \sin(t^2)\, dt$.

(a) Find $A'(x)$.

(b) Find the x-value of the first positive critical number of $A(x)$.

(c) At that first positive critical number you found in (b), does $A(x)$ have a local max, a local min, or neither? Justify your answer.

13. You are bored so you begin to pour the salt out of a salt shaker at a constant rate of 3 cubic inches per second onto the table in such a way that it forms a conical pile whose height is always half the radius of the base. How fast is the base radius changing when the radius is 2 inches? Include units. (Recall that for a cone, $V = \frac{1}{3}\pi r^2 h$.)