Please do not write in these boxes.

1	2	3	4	5	6	7	8	9	10	11	12	13	TOTAL

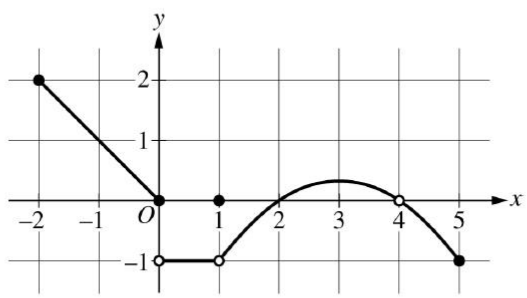
MTH 1321 Name:____________________

Fall 2018 Departmental Final Exam Instructor:____________________

NO CALCULATOR ALLOWED

Complete the following in the space provided. Show the steps leading to your answer for full credit.

1. Consider the graph of f shown below.



Graph of f

(a) Find all values of a in $[-2, 5]$ such that $\lim_{x \to a} f(x) = 0$.

(b) Is f continuous at $x = 1$? Explain why or why not.

(c) Find the average rate of change of f on the interval $[-2, 0]$.

(d) Find $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$. Justify your answer.

2. Compute the following:

   (a) Let $y = \sin^{-1}(3x) + 2^x + \log_{10} x$. Find $y'(x)$. You do not need to simplify your answer.

   (b) Let $y = \sqrt{1 + e^{-x}}$. Find $\dfrac{dy}{dx}$. You do not need to simplify your answer.

3. Compute the following:

   (a) Let $y = \dfrac{\sec x}{\ln x} + x^{-1}\cos(2x)$. Find $y'(x)$. You do not need to simplify your answer.

   (b) Let $x^2 + xy - 3y = 3$. Find $\dfrac{dy}{dx}$ at the point $(2, 1)$.

4. Researchers are studying the effectiveness of a new antibiotic. Let $P(t)$ be the number of bacteria present t hours after the antibiotic is administered. Data collected by the researchers indicated that the over the first 12 hours, the number of bacteria decreased while its rate of change increased, and then afterwards the number of bacteria continued to decrease, but its rate of change decreased.

(a) Which first derivative statements best describe this scenario?
(A) $P'(t) > 0$ for $0 < t < 12$ and $P'(t) > 0$ for $t > 12$
(B) $P'(t) > 0$ for $0 < t < 12$ and $P'(t) < 0$ for $t > 12$
(C) $P'(t) < 0$ for $0 < t < 12$ and $P'(t) > 0$ for $t > 12$
(D) $P'(t) < 0$ for $0 < t < 12$ and $P'(t) < 0$ for $t > 12$
(E) None of the above

(b) Which second derivative statements best describe this scenario?
(A) $P''(t) > 0$ for $0 < t < 12$ and $P''(t) > 0$ for $t > 12$
(B) $P''(t) > 0$ for $0 < t < 12$ and $P''(t) < 0$ for $t > 12$
(C) $P''(t) < 0$ for $0 < t < 12$ and $P''(t) > 0$ for $t > 12$
(D) $P''(t) < 0$ for $0 < t < 12$ and $P''(t) < 0$ for $t > 12$
(E) None of the above

(c) At $t = 12$, the graph of $y = P(t)$ has
(A) a local maximum
(B) a local minimum
(C) a critical number but no local extremum
(D) an inflection point
(E) None of the above

(d) Assuming that there are a positive number of bacteria, P_0, present at time $t = 0$ and that at time $t = 24$, all bacteria are eliminated, sketch a possible graph of P corresponding to all the criteria described above.

5. Consider $f(x) = \sqrt{x}$.

   (a) Find the linear approximation of $y = f(x)$ at $x = 4$.

   (b) Let R denote the region between $y = f(x)$ and the x-axis from $x = 0$ to $x = 1$. The vertical line $x = k$ divides R into two regions of equal area. Find k.

6. A rectangle has base x cm and height y cm. The base is increasing at a rate of 2 cm/sec and the height is decreasing at a rate of 1 cm/sec. Find the rate of change of the area of rectangle (with respect to time) when $x = 7$ and $y = 3$.

7. Suppose $f'(x) = x(x-1)^2$.

   (a) Find the x-coordinate for all local extrema of f. Classify each as where a local max or local min occurs.

   (b) Find the x-coordinate for all inflection points of f.

8. Suppose that the tangent line to $y = f(x)$ at $x = 2$ is given by $\ell(x) = 3(x-2)+4$. Mark each of the following statements as true (T) or false (F). You do *not* need to justify your answer.

   (a) f is concave up at $x = 2$

   (b) $\int_0^1 \ell(x)\, dx > 0$

   (c) $\lim_{x\to\infty} \frac{\ell(x)}{2x+3} = \infty$

   (d) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

   (e) $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 3$

9. Compute the following:

(a) $\displaystyle\int \left(\frac{1}{x} + \frac{1}{1+x^2} - e^x\right) dx$

(b) $\displaystyle\int \frac{\ln x}{x}\, dx$

10. Compute the following:

(a) $\displaystyle\int_0^{\pi/4} \cos x \, dx$

(b) $\displaystyle\int_1^3 \frac{x}{x^2+1}\, dx$

11. The acceleration of a particle at time t is given by $a(t) = t - \sin t$. At time $t = 0$, the particle's position is $s(0) = -2$ and the particle's velocity is $v(0) = 2$. Find $s(t)$, the position of the particle at any time $t \geq 0$.

12. While you are exercising, data regarding the rate at which you are burning calories is collected. Selected values are shown in the table below.

t (min)	0	10	30	60
$r(t)$ (calories/min)	13	11	8	7

Suppose $y = r(t)$ is differentiable on $0 \leq t \leq 60$.

(a) Estimate $r'(20)$. Include units.

(b) Use a left Riemann sum with the three subintervals indicated in the table to estimate the total number of calories burned during the 60 minute workout.

(c) Was there some time during your workout when you were burning 10 calories per minute? Explain why or why not.

13. Suppose Lake Waco contains 100 million cubic meters of water at time $t = 0$. During the time interval $0 \leq t \leq 4$ hours, a rainstorm occurs which causes water to enter the lake at a rate modeled by $E(t) = 16t - 3t^2$, where $E(t)$ is measured in millions of cubic meters per hour. During the same time interval, the gates on the dam are opened to release water out of the lake at a rate modeled by $R(t) = -2t + 15$, where $R(t)$ is measured in millions of cubic meters per hour.

(a) Write an expression involving one or more integrals for the total amount of water (in millions of cubic meters) in the lake at time t for $0 \leq t \leq 4$.

(b) Find the total amount of water in the lake at $t = 4$.

(c) Find the minimum amount of water in the lake for $0 \leq t \leq 4$. Justify your answer.