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DO NOT WRITE IN THESE BLOCKS

NAME _____

MTH 1322 - CALCULUS II

INSTRUCTOR _____

DIRECTIONS: Box your final answers. You must show enough of your work so that the reader can follow what you did. If it is possible to find an exact answer by taking an algebraic approach, you may not receive full credit for an approximation. This means that whenever possible, write your final answers in exact form (π , $\sqrt{3}$, $\ln 2$, etc.) and not in decimal form (3.14159, 1.73205, 0.693147, etc.)

1. Use integration by parts to evaluate $\int_0^{\pi} x \sin(2x) dx$.

2. Use a trigonometric substitution to evaluate $\int \frac{x^2 dx}{\sqrt{4-x^2}}$.

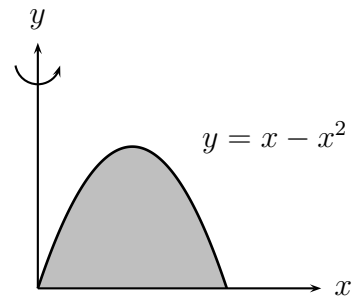
3. Use a partial fraction decomposition to evaluate $\int \frac{dx}{x^2(x+1)}$.

4. Determine if the improper integral converges and, if so, find its value.

(a) $\int_0^{\infty} e^{-x} dx$

(b) $\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}}$

5. Use the most convenient method to find the volume of the solid that is obtained by rotating the region between $y = x - x^2$ and $y = 0$ about the y -axis.



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6. Solve the initial value problem $y' = -\frac{x}{y}$, $y(0) = -2$.

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7. Write the geometric series

$$\frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} + \dots$$

in Σ -notation and find its sum.

8. Determine whether the series converges or diverges. Clearly state which tests you are using.

(a)
$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

(b)
$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + 1}}$$

9. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$. Pay attention to the endpoints.

10. (a) Find the Taylor polynomial $T_2(x)$ centered at $a = 3$ of the function $f(x) = \ln x$.

(b) Suppose $g(x)$ is a function that has a Maclaurin series of the form

$$1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \dots .$$

Find the value of $g'''(0)$.

11. Find the value of $\int_0^{\frac{1}{2}} e^{-x^3} dx$ to within an error of at most 10^{-3} .

(*Hint:* Use the Maclaurin series of e^{-x^3} to write the integral as an alternating series.)

12. (a) Use De Moivre's Theorem to write $(\sqrt{3} + i)^6$ in the form $a + bi$.

(b) Write $e^{-i\pi/4}$ in the form $a + bi$.

Table of Trigonometric Integrals

$$\int \tan x \, dx = \ln |\sec x| + C = -\ln |\cos x| + C$$

$$\int \cot x \, dx = -\ln |\csc x| + C = \ln |\sin x| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$\int \csc x \, dx = -\ln |\csc x + \cot x| + C$$

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$$

$$\int \cot^n x \, dx = -\frac{1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x \, dx$$

$$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$$\int \csc^n x \, dx = -\frac{1}{n-1} \csc^{n-2} x \cot x + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx$$