INSTRUCTIONS:

Turn off your phone. No references. No collaboration. Prove four theorems with at least one theorem from each section.

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1 General Topology.

Theorem 1.1 For a topological space X, the following are equivalent:

a. X is compact;

b. each family, $\mathcal{E}$ of closed subsets of X with the finite intersection property has nonempty intersection;

c. each net has a cluster point;

d. each filter has a cluster point;

e. each ultranet converges;

f. each ultrafilter converges.

Theorem 1.2 a. Suppose that $X = \bigcup_{\alpha \in A} X_\alpha$, and suppose that each X_α is connected and $\bigcap_{\alpha \in A} X_\alpha \neq \emptyset$. Then X is connected.

b. If each pair, x, y, of points of X lies in some connected subset E_{xy} of X, then X is connected.

c. If E is a connected subset of X and $E \subseteq A \subseteq \overline{E}$ then A is connected.

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2 Homotopy

Theorem 2.1 Let $p : E \to B$ be a covering map, and let $p(e_0) = b_0$. Any path $f : [0, 1] \to B$ beginning at b_0 has a unique lifting to a path $\tilde{f}$ in E beginning at e_0.

Theorem 2.2 Suppose that $X = U \cup V$ where U and V are open and $U \cap V$ is nonempty and path connected. If U and V are simply connected then so is X.

Theorem 2.3 a. The fundamental group of S^1 is $\mathbb{Z}$.

b. The fundamental group of the figure eight is not abelian.

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3 Homology

Theorem 3.1 a. Let T be the torus. Then $H_1(T) \simeq \mathbb{Z} \oplus \mathbb{Z}$ and $H_2(T) \simeq \mathbb{Z}$.

b. Let S be the Klein bottle. Then $H_1(S) \simeq \mathbb{Z} \oplus \mathbb{Z}/2$ and $H_2(T) \simeq 0$.

Theorem 3.2 Let $f, g : K \to L$ be simplicial maps.

a. If there is a chain homotopy between $f_\sharp$ and $g_\sharp$ then the induced homomorphisms f_* and g_* are equal.

b. If f and g are contiguous then there is a chain homotopy between $f_\sharp$ and $g_\sharp$.

Theorem 3.3 State and prove the Zig-Zag Lemma.

I swear that all the work handed in was performed by me alone with no reference to any books or notes or any outside help.

Signed: