In this research project, we shall continuously work on the theory of quantum gravity at fixed Lifshitz points, proposed by Horava in 2009. Since then it has attracted lot of attention, and several major problems found so far, such as ghost, instability, strong coupling, and different speeds in the gravitational sector, are well-understood and under control in several modified versions of the theory, including the ones with the local $U(1)$ symmetry both with and without the projectability condition. In this research proposal, we shall investigate several important issues of the theory, and apply it to cosmology and astrophysics. The purposes of the latter are two-folds: (a) Check the consistency of the theory. If the theory properly describes the reality of gravity, it should be consistent with observations. (b) Find distinguishable features of the theory, so it can be tested by observations.

In particular, we shall investigate the effects of the Lorentz symmetry violation in the gravitational and matter sectors, stability and quasi-normal modes of black holes and stars, constraints of the theory from solar system tests, the interacting models of dark matter and dark energy, primordial gravitational waves, and inflationary scenario.