Hello! This resource focuses on material covered in the tenth week of classes, namely topics from sections 4.3 - 4.4 of OpenStax’s Precalculus. Please refer to other resources if your class is at a different point.

Don’t forget to come to Group Tutoring on Tuesdays at 6:30pm in room 74 of the basement of Sid Rich!

Keywords: Logarithmic Functions and their graphs

Topic of the Week:
Log Functions

Last week, we discussed exponential functions, and you may be wondering, “What is the inverse of an exponential function?” The inverse of the exponential function \( f(x) = b^x \) is \( f^{-1}(x) = \log_b(x) \). This function is called a logarithm or a logarithmic function, and it has the following definition:

\[
y = \log_b(x) \text{ is equivalent to } b^y = x
\]

under the following conditions:

- The input \( x \) must be greater than zero.
- The base \( b \) must be greater than zero (as with exponential functions).
- The base \( b \) must NOT be equal to 1 (as with exponential functions).

In words, the logarithm (or log) \( y \) with base \( b \) of \( x \) is “the exponent to which \( b \) must be raised to get \( x \).” Also, because the logarithm is the inverse of the exponential, its domain is the range of the exponential, and its range is the exponential’s domain.

- Domain: \((0, \infty)\)
- Range: \((-\infty, \infty)\)

Because logarithms are equivalent to exponentials, we can convert logarithms to exponentials and vice versa. Rewriting a logarithm in exponential form makes it easier to evaluate a logarithm because we can ask ourselves, “To what exponent should \( b \) be raised in order to get \( x \)?”

There are two logarithms that are frequently used and have special names:

- A common logarithm has a base of 10 and is often written \( \log(x) \). If you see a logarithm without a base, it is implied that it is a common or base-10 logarithm.
- A natural logarithm has a base of \( e \) and is written \( \ln(x) \).
Highlight #1: Graphs of Logarithmic Functions

Finding a Logarithm’s Domain

If a logarithm’s argument (the expression inside the parentheses) is not simply \( x \), the domain will be restricted based on that argument. Because the logarithm is undefined for input values less than or equal to zero, we can find the domain of a logarithmic function through the following steps:

1. Set the expression inside the parentheses (argument) greater than zero.
2. Solve the resulting inequality for \( x \).

Graphing a Logarithmic Function

The parent logarithmic function, \( f(x) = \log_b(x) \), is graphed as shown below. Note these features:

- a vertical asymptote at \( x = 0 \)
- an \( x \)-intercept at (1, 0)
- a “key point” at \((b, 1)\)

We can see that if the base is greater than 1, the graph will be increasing, while if the base is between zero and one, the graph will be decreasing. We can verify this by looking at the equivalent exponential \( b^y = x \). If we have a base of \( b = 2 > 1 \), we must increase the \( y \) values to get greater \( x \) values. However, if we have a base of \( b = \frac{1}{2} < 1 \), we must use more negative \( y \) values to get greater \( x \) values. For example,

\[
\left(\frac{1}{2}\right)^2 = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4
\]

We must have a negative exponent (\( y \) value) to change the fraction into a number greater than 1.
To draw a graph of the parent logarithm, we can follow the process shown below.

All other logarithmic functions can be graphed as transformations of the parent logarithmic function. Note that a logarithm that is horizontally shifted, like \( f(x) = \log(x - c) \), will have the vertical asymptote \( x = c \). A horizontal stretch/compression can also affect the vertical asymptote.

**Things you might struggle with:**
- Make sure when you rearrange from logarithmic form to exponential form and vice versa that you properly put the right values where they belong.
- Remember that all \( \ln \) function is logarithm in base \( e \).
- Logarithms are especially difficult because they are hard to picture and find a use for at this stage, but I promise they will get better with more practice!

**Example Problems**

1. Rewrite the equation \( y = \ln(x) \) in exponential form.

2. Solve the equation \( \log_3(x) = -2 \) for \( x \) by rewriting it in exponential form.

3. Find the domain of the logarithmic function \( f(x) = \log_2(-3x + 9) + 5 \).
**Example Problem Answers**

1. Using the log to exp equation, we can assign values to each of the variables! The trick here is that ln is simply base e.

   \[ e^x = y \]

2. We have to convert this equation to exponent form, and it’ll be an easy solve!

   \[ x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \]

3. We know that we cannot take a log of a negative nor zero value, so everything in those parentheses has to be larger than 0.

   \[-3x + 9 > 0 \]
   \[-3x > -9 \]
   \[ x < 3 \]

   So the domain is \((-\infty, 3)\).

**Reminders**

- Group Tutoring for Precalculus is on Tuesday evenings from 6:30-7:30 in room 74 of Sid Rich! This session is a great time to come ask specific questions and work together through some problems.

- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at [https://www.baylor.edu/case/index.php?id=978622](https://www.baylor.edu/case/index.php?id=978622)

- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at [https://www.baylor.edu/tutoring](https://www.baylor.edu/tutoring)

All diagrams, tables, and external information is property of Precalculus by J. Abramson et al. unless otherwise specified.

Available: https://openstax.org/details/books/precalculus