Howdy. This resource focuses on material covered in the ninth week of classes, namely topics from sections 6.1 – 6.2 of OpenStax’s Precalculus. Please refer to other resources if your class is at a different point. Don’t forget to come to Group Tutoring on Mondays at 5:15 in room 75 of the basement of Sid Rich!

**Keywords:** *Graphs of Trigonometric Functions*

**Topic of the Week:**

**Graphs of Trig Functions**

Before we look at graphs of the sine and cosine functions, let’s review some important terms.

The trigonometric functions are periodic, which means that they are functions “for which a specific horizontal shift, \( P \), results in a function equal to the original function”’. In other words, periodic functions repeat over and over. Figure 1 shows an example of a periodic function that is not one of the trigonometric functions.

![Figure 1. A Periodic Function](image)

The horizontal shift \( P \) is called the period, and it is “the shortest interval over which a function completes one full cycle”. In equation form, a periodic function \( f(x) \) adheres to the following rule for all \( x \) values in its domain:

\[
 f(x + P) = f(x).
\]
Highlight #1:

In contrast to the sawtooth function in Figure 1, the sine and cosine functions oscillate smoothly like waves. See Figure 2 and Figure 3 for graphs of the sine and cosine functions.

Note that they both have a period of \(2\pi\). See Figure 4 for other characteristics of sine and cosine.

**Characteristics of Sine and Cosine Functions**

The sine and cosine functions have several distinct characteristics:

- They are periodic functions with a period of \(2\pi\).
- The domain of each function is \((-\infty, \infty)\) and the range is \([-1, 1]\).
- The graph of \(y = \sin x\) is symmetric about the origin, because it is an odd function.
- The graph of \(y = \cos x\) is symmetric about the \(y\)-axis, because it is an even function.

**Sinusoidal Functions**

A function that can be described as a combination of transformations of the sine or cosine function is called a **sinusoidal function** or simply a **sinusoid**. Sinusoids have the general form

\[ y = A \sin(Bx - C) + D \quad \text{or} \quad y = A \cos(Bx - C) + D \]

Because cosine and sine are shifted versions of each other, the equation for a sinusoid can be written with cosine or sine. You can check if two equations for a sinusoid are equivalent using the cofunction identities from section 5.4.

In addition to the period, which is \(2\pi/|B|\), sinusoids have a few other notable characteristics:

- **midline** – the horizontal line through the middle of the graph, \(y = D\)
- **amplitude** – “the vertical height from the midline”, \(|A| \quad [1]\)
- **phase shift** – “the horizontal displacement of the basic sine or cosine function”, \(C/B\)
An example sinusoid with its midline, amplitude, and period labeled is shown in Figure 5. The phase shift is not shown because it changes based on whether we use a sine or cosine function to describe the wave. If we use a sine function, the phase shift is zero, but if we use a cosine function, the phase shift is $\pi/4$.

To graph a sinusoid, follow these steps:

1. If necessary, write the function in the form $y = A \sin (Bx - C) + D$ or $y = A \cos (Bx - C) + D$.
2. Determine its amplitude, $|A|$.
3. Determine its period, $P = \frac{2\pi}{|B|}$.
4. Graph the function $y = A \sin(Bx)$ or $y = A \cos(Bx)$.
5. Shift the graph of $y = A \sin(Bx)$ or $y = A \cos(Bx)$ left or right according to the phase shift, $C/B$.
6. Shift the graph from step 5 up or down according to the vertical shift, $D$.
7. If $A$ is negative, reflect the graph across the $x$-axis. ($B$ is not usually negative, but a negative $B$ value would cause a reflection across the $y$-axis.)

To graph $y = A \sin(Bx)$ for step 4 above, follow these steps:

1. Plot a point at the origin.
2. Plot points on the $x$-axis at $x = \pm P/2, \pm P, \pm 3P/2, \pm 2P, \ldots$
3. Plot points with the $y$-value $\pm A$ at $x = \pm P/4, \pm 3P/4, \pm 5P/4, \pm 7P/4, \ldots$ Start with $A$ and alternate back and forth with $-A$. (If $A$ is positive, you will get a peak first, and if $A$ is negative, you will get a trough first.)
4. Draw a sinusoidal curve through the points.

To graph $y = A \cos(Bx)$ for step 4 above, follow these steps:

1. Plot a point $(0, A)$. (If $A$ is positive, this will be a peak, and if $A$ is negative, this will be a trough.)
2. Plot points with the $y$-value $\pm A$ at $x = \pm P/2, \pm P, \pm 3P/2, \pm 2P, \ldots$ Start with $-A$ and alternate back and forth with $A$.
3. Plot points on the $x$-axis at $x = \pm P/4, \pm 3P/4, \pm 5P/4, \pm 7P/4, \ldots$
4. Draw a sinusoidal curve through the points.
Highlight #2: Graphs of the Other Trig Functions

Just like sine and cosine, the other trigonometric functions repeat. The graphs of tangent and cotangent are very similar because they are reciprocal functions. See Figure 6 and Figure 7.

Note that tangent and cotangent’s periods are $\pi$, and cotangent has asymptotes where tangent has zeros and vice versa. Also, the parent tangent function crosses through $y = \pm 1$ at $x = \pm \pi/4$.

The graphs of secant and cosecant are also very similar because they are reciprocal functions of cosine and sine, respectively, and cosine and sine are very similar. See Figure 8 and Figure 9.

Secant’s asymptotes occur where cosine is zero and cosecant’s asymptotes occur where sine is zero.

Transformations of tangent, cotangent, secant, and cosecant can be graphed in a similar manner to transformations of sine and cosine.
Things you might struggle with:

1. Sine and Cosine are exactly the same graph, but with a phase shift! Basically, it means that they are the same moving left and right.
2. The unit circle the sin/cos graphs are heavily correlated. Try and figure out how these correspond to each other!

Example Problems

No examples this week! All concepts.

If you haven’t memorized the unit circle yet, this is the time to do so! After this is memorized, see how it relates to the graphs of sine and cosine.

Example Problem Answers

Reminders

- Group Tutoring for Precalculus is on Monday evenings from 5:15 to 6:15 in room 75 of Sid Rich! This is a great time to come ask specific questions to work together through some problems.
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/case/index.php?id=978622
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at https://www.baylor.edu/tutoring

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