MTH 1320: Pre-Calc Resource

Week of 3/28/2022

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Howdy. This resource focuses on material covered in the eleventh week of classes, namely topics from sections 4.5 - 4.6 of OpenStax’s Precalculus. Please refer to other resources if your class is at a different point. Don’t forget to come to Group Tutoring on Mondays at 5:15 in room 75 of the basement of Sid Rich!

Keywords: Logarithmic Properties, Exponential and Logarithmic Equations

Topic of the Week:

Log Properties

When working with exponentials and logarithms, it is important to know their properties. Your textbook does not have a separate section covering properties of exponential functions, so the following table summarizes both exponential and logarithmic properties. Because exponentials and logarithms are inverses, it is not surprising that their properties also look inverted.

<table>
<thead>
<tr>
<th>Name of Property</th>
<th>Exponential Version</th>
<th>Logarithmic Version</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b^0 = 1$</td>
<td>$\log_b(1) = 0$</td>
</tr>
<tr>
<td></td>
<td>$b^1 = b$</td>
<td>$\log_b(b) = 1$</td>
</tr>
<tr>
<td>Inverse property</td>
<td>$b^{\log_b x} = x$ ($x &gt; 0$)</td>
<td>$\log_b b^x = x$</td>
</tr>
<tr>
<td>One-to-one property</td>
<td>$b^x = b^y$ if and only if $x = y$</td>
<td>$\log_b(M) = \log_b(N)$ if and only if $M = N$</td>
</tr>
<tr>
<td>Product rule</td>
<td>$b^x b^y = b^{x+y}$</td>
<td>$\log_b(MN) = \log_b(M) + \log_b(N)$</td>
</tr>
<tr>
<td>Quotient rule</td>
<td>$\frac{b^x}{b^y} = b^{x-y}$</td>
<td>$\log_b \left( \frac{M}{N} \right) = \log_b(M) - \log_b(N)$</td>
</tr>
<tr>
<td>Power rule</td>
<td>$(b^x)^y = b^{xy}$</td>
<td>$\log_b(M^n) = n \log_b(M)$</td>
</tr>
</tbody>
</table>

Highlight #1: Expanding and Condensing Logarithmic Expressions

Logarithmic properties can be used to expand or condense logarithmic expressions. To expand a logarithmic expression, you should often use the following order:

1. Quotient rule
2. Product rule
3. Power rule
Note that when you have a ratio of products, an alternate approach is to write every factor in the numerator with a plus sign and every factor in the denominator with a minus sign, as follows:

\[ \log_b \left( \frac{M_1 M_2 \ldots M_M}{N_1 N_2 \ldots N_N} \right) = \log_b(M_1) + \log_b(M_2) + \cdots + \log_b(M_m) - \log_b(N_1) - \log_b(N_2) - \cdots - \log_b(N_n) \]

To condense a logarithmic expression, you should use the reverse of the expansion order:

1. Power Rule
2. Product Rule
3. Quotient Rule

Highlight #2: Change of Base Formula

Another useful tool is the change of base formula which says

\[ \log_n(M) = \frac{\log_b(M)}{\log_b(n)} \]

where \( n \) is a new base. The change of base formula can be used to evaluate logarithms in a calculator that only has functions for the common logarithm (\( \log(x) \)) and the natural logarithm (\( \ln(x) \)).

Highlight #3: Exponential and Logarithmic Equations

Now that you know the exponential and logarithmic properties, you can use them to solve exponential and logarithmic equations.

There are two main ways to solve exponential equations. Before we discuss them, note that a positive base raised to a power cannot produce a negative number, and a logarithm cannot have a negative argument. Therefore, some exponential equations might have no solution or an extraneous solution, “a solution that is correct algebraically but does not satisfy the conditions of the original equation”.

Because of the one-to-one property of exponential functions, we know that if two exponentials are equal and have the same base, their exponents are also equal. We can use this property to solve exponential equations when the exponentials have the same base. If the exponentials do not have the same base, we can sometimes rewrite one of the bases as a power of the other base.

Highlight #4: Using Logarithms

If the exponentials cannot be written with the same base or there is only one exponential term, we can still solve the equation using logarithms. Equality is maintained if we take the same logarithm of both sides of the equation. Your textbook recommends using the common logarithm if one of the exponentials has 10 as its base and using the natural logarithm otherwise. Once you have applied the logarithm to both sides of the equation, you can use the properties of logarithms to solve.
There are also two main ways to solve logarithmic equations. **Note that logarithmic equations may have extraneous solutions as well, which cause the argument of a logarithm to be negative.**

Recall that a logarithm has the following definition:

\[ y = \log_b(x) \text{ is equivalent to } b^y = x \]

We can often use this fact to solve logarithmic equations by rewriting a logarithm in its exponential form. **Another way to look at this is that, just like we can take the logarithm of each side of an equation, we can “exponentiate” each side of an equation and maintain equality.**

Just like for exponentials, if we have an equation with two logarithms of the same base, we can use the one-to-one property to solve it. **We can set the arguments of the two logarithms equal to each other if the two logarithms are equal.**

**Things you might struggle with:**

1. There are many different ways to solve these problems! Just practice.
2. Multiplication is addition and exponentials are multiplication when you think about it hard. Additionally, division is subtraction and negative exponents are division.
3. Don’t forget that any number to the power of 0 is 1!

**Example Problems**

1. Expand the following:

\[ \ln \left( \frac{\sqrt{(x-1)(2x+1)^2}}{x^2-9} \right) \]

2. Solve \(5^x = 25^{3x + 2}\)

3. Solve \(2^x = 3^{x+1}\)

4. Solve \(2 \ln(x + 1) = 10\)

5. Solve \(\ln(x^3) = \ln(1)\)
Example Problem Answers

1. Use all the rules put together!

Use the quotient rule for logarithms:

\[
\ln\left(\frac{(x - 1)(2x + 1)^2}{(x^2 - 9)}\right) = \ln\left(\sqrt{(x - 1)(2x + 1)^2}\right) - \ln(x^2 - 9)
\]

Rewrite the square root as an exponent:

\[
= \ln\left(\left((x - 1)(2x + 1)^2\right)^{1/2}\right) - \ln(x^2 - 9)
\]

Use the power rule of exponents in the first logarithm:

\[
= \ln\left((x - 1)\left(\frac{1}{2}(2x + 1)^1\right)\right) - \ln(x^2 - 9)
\]

Factor the argument of the second logarithm:

\[
= \ln\left((x - 1)\left(\frac{1}{2}(2x + 1)^1\right)\right) - \ln((x - 3)(x + 3))
\]

Use the product rule for logarithms:

\[
= \ln\left((x - 1)^{\frac{1}{2}}\right) + \ln(2x + 1) - [\ln(x - 3) + \ln(x + 3)]
\]

Distribute the negative sign:

\[
= \ln\left((x - 1)^{\frac{1}{2}}\right) + \ln(2x + 1) - \ln(x - 3) - \ln(x + 3)
\]

Use the power rule for logarithms on the first logarithm:

\[
= \frac{1}{2}\ln(x - 1) + \ln(2x + 1) - \ln(x - 3) - \ln(x + 3)
\]

2. Let’s solve this one using power rules

Rewrite 25 as a power of 5:

\[
5^{2x} = (5^2)^{3x+2}
\]

Use the power rule of exponents:

\[
5^{2x} = 5^{2(3x+2)}
\]

Distribute the 2:

\[
5^{2x} = 5^{6x+4}
\]

Use the one-to-one property of exponential functions:

\[
2x = 6x + 4
\]

Combine like terms and solve for x:

\[
-4x = 4
\]

\[
x = -1
\]
3. Let’s use log rules for this!
Take the natural logarithm of both sides of the equation:
$$\ln(2^x) = \ln(3^{x+1})$$
Use the power rule for logarithms:
$$x\ln(2) = (x + 1)\ln(3)$$
Distribute the $\ln(3)$:
$$x\ln(2) = x\ln(3) + \ln(3)$$
Combine like terms and solve for $x$:
$$x\ln(2) - x\ln(3) = \ln(3)$$
$$x(\ln(2) - \ln(3)) = \ln(3)$$
$$x = \frac{\ln(3)}{\ln(2) - \ln(3)}$$
Use the quotient rule for logarithms to simplify:
$$x = \frac{\ln(3)}{\ln\left(\frac{2}{3}\right)}$$

4. Isolate the values you need!
Isolate the natural logarithm:
$$\ln(x + 1) = \frac{10}{2}$$
$$\ln(x + 1) = 5$$
Use the definition of the natural logarithm:
$$x + 1 = e^5$$
Isolate the $x$:
$$x = e^5 - 1$$

5. Use log properties!
Use the one-to-one property of logarithms:
$$x^2 = 1$$
Take the square root of both sides of the equation:
$$\sqrt{x^2} = \pm\sqrt{1}$$
$$x = \pm 1$$
Check for extraneous solutions:
$$\ln(1^2) = \ln(1) = 0$$
$$\ln((-1)^2) = \ln(1) = 0$$
Both 1 and $-1$ are valid solutions.
Reminders

- Group Tutoring for Precalculus is on Monday evenings from 5:15 to 6:15 in room 75 of Sid Rich! This is a great time to come ask specific questions to work together through some problems.
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/case/index.php?id=978622
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at https://www.baylor.edu/tutoring

All diagrams, tables, and external information is property of Precalculus by J. Abramson et al. unless otherwise specified.

Available: https://openstax.org/details/books/precalculus