Hello everybody, my name is Nathaniel Takle and I will be the Calculus Master tutor this year. I am here to help you excel in Calculus throughout the semester! My biggest tip for learning calculus is doing practice problems! If you are stumped by a practice problem, refer to videos that work the problem and explain the concept! The following is a link to the Baylor Tutoring YouTube page with videos on EACH concept that is covered during Calculus! (https://www.baylor.edu/case/index.php?id=978621) I would love to help you succeed in any way, please feel free to reach out to me at Nathaniel_Takle1@baylor.edu if you have questions or would like further explanation on a topic.

These resources are a culmination of the main topics learned each week. These are meant to provide you with explanations and more practice to master Calculus this semester!! Remember: The Tutoring Center offers free individual and group tutoring for Calculus.

**Calculus Group Tutoring sessions will be Mondays from 5:15-6:15 PM at the Sid Rich basement, room 74!**

You can reserve a spot at https://baylor.edu/tutoring. I hope to see you there!

**Keywords**: Related Rates, Linear Approximation, & Critical Points

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**Topic of the Week: Related Rates**

Chapter 3.10

- WHOOP WHOOP! The infamous related rates. This is generally one of the most difficult sections of the book for students to learn, and as a result the professor may devote extra time to going over it. Despite how difficult it is, at its core Related Rates problems are very simple. We are going to master them. The easiest way to solve a Related Rates problem is by drawing a picture, determining exactly what you are solving for, then figuring out from your picture a formula that allows you to put what you want into terms of what you already know. This is made more difficult because you are often given the value of a mix of both constants and derivatives, so using implicit differentiation to get what you want into terms of what you have is key.

- Video Resources
Example Problem 1

- A ladder is against a building and slowly sliding down. Let $h(t)$ be the height of the top of the ladder to the ground at time $t$, let $x(t)$ be the distance between the base of the building and the base of the ladder at time $t$, the ladder is 5 meters long, $x(0)=1.5$, and the speed at which the base of the ladder is moving away from the building is $.8 \text{ m/s}$. Find the speed at which the height of the ladder ($h(x)$) is changing at $t=1$.
- TRY AND SOLVE THIS USING THE STEPS LISTED ABOVE. The answer process is shown below so you can check your work.

  - What are we looking for?
    - $\frac{dh(t)}{dt}$ at $t = 1$
  - What do we know
    - $x(0)=1.5$
    - $x(t) = mt + x(0)$
    - $m = \frac{dx}{dt}$
    - $\frac{dx}{dt} = .8$
    - $h(t)^2 + x(t)^2 = 5^2$
    - $\frac{d}{dt} h^2 + \frac{d}{dt} x^2 = \frac{d}{dt} 5^2$ (Implicit differentiation)
    - \[ 2h \frac{dh}{dt} + 2x \frac{dx}{dt} = 0 \]
    - $\frac{dh}{dt} = -\frac{2x}{2h} \frac{dx}{dt}$
  - From here, find $x(1)$, $h(1)$, and plug them and $\frac{dx}{dt} = .8$ into the formula we just created to find the answer.
  - Answer: $\frac{dh}{dt} = -.41$

Example problem 2

- A 5.5ft person is walking away from a lamp at a speed of 2ft/second. A 12ft lamp is behind him casting a shadow in front of him. Let $x(p)$ be the distance from the lamp to the person at time $t$, let $x(s)$ be the distance of the shadow at time $t$. How fast is the tip of the shadow moving away from the person?

HINT: They are similar triangles... (this can be an equation)
**Highlight: Linear Approximation**

Chapter 4.1
- In this chapter the book introduces linear approximations of functions. The linear approximation of a function is defined as such:
  - \[ L(x) = f(a) + f'(a)(x - a), \text{ with } L(x) \approx f(x) \]
  - In differential notation we treat \( \Delta y \approx dy \) and \( \Delta x \approx dx \) for a small \( dy \) and \( dx \).
- The error and percentage error for of our Linear Approximation is defined as
  1) \( \text{error} = |\Delta f - f'(a)\Delta x| \)
  2) \( \% \text{error} = \left| \frac{\text{error}}{\text{actual value}} \right| \times 100\% \)
- Example Problem 3
  - Estimate \( x^3 \) from the point \( x=2 \) to the point \( x=2.03 \). What is the % error of your estimation?

**Highlight: Critical Points**

Chapter 4.2
- In this chapter the classes are going over finding local maximums and minimums using Calculus by finding critical points, then observing how the function looks at either side of the point. When \( f'x = 0 \), there is not slope, no rate of change. Thus, when it equals zero, the tangent line is flat which can indicate a change from increasing to decreasing on a parent graph. If the slope to the left of the critical point is positive and to the right is negative, the graph changes from increasing to decreasing (concave down). See graph.
- Video Resources
  - [https://www.youtube.com/watch?v=nMsn8Txx8to&feature=youtu.be](https://www.youtube.com/watch?v=nMsn8Txx8to&feature=youtu.be)
- Example Problem
  - What is are the critical points of \( f(x) = \frac{x^2+10x+25}{x+5} \)? Are they maximums or minimums?
Topics Commonly Struggled with:

- Related Rates
- Determining equations within related rates (brush up on equation/volume/circumference of different shapes [triangles, cylinder, sphere, cone, etc.])

Example Problems:

1) A ladder is against a building and slowly sliding down. Let \( h(t) \) be the height of the top of the ladder to the ground at time \( t \), let \( x(t) \) be the distance between the base of the building and the base of the ladder at time \( t \), the ladder is 5 meters long, \( x(0)=1.5 \), and the speed at which the base of the ladder is moving away from the building is .8 m/s. Find the speed at which the height of the ladder (\( h(x) \)) is changing at \( t=1 \).

2) A 5.5ft person is walking away from a lamp at a speed of 2ft/second. A 12ft lamp is behind him casting a shadow in front of him. Let \( x(p) \) be the distance from the lamp to the person at time \( t \), let \( x(s) \) be the distance of the shadow at time \( t \). How fast is the tip of the shadow moving away from the person?

HINT: They are similar triangles...

(this can be an equation)

3) Estimate \( x^3 \) from the point \( x=2 \) to the point \( x=2.03 \). What is the % error of your estimation?
4) What are the critical points of \( f(x) = \frac{x^2 + 10x + 25}{x+5} \)? Are they maximums or minimums?
1. Answer: $\frac{dh}{dt} = \mathbf{-.411}$
2. Answer: $x's = 1.692 \text{ or } \frac{22}{13}$
3. Answer = $L(2.03) = 8 + (12)(0.03) = 8.36$
   - % error = $\left| \frac{8.3654 - 8.36}{8.364 - 8.36} \right| \times 100\% = 0.06\%$
4. Answer = Critical points are at $x = -2$ (local max), $-8$ (local min), and $-5$ (neither).