Physics 1408/1420

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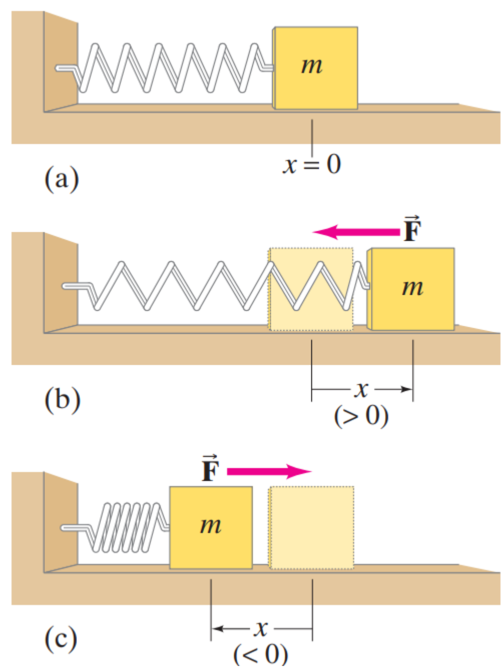
Hello Fellow Physicists

I am Aman Patel, the Master Tutor for Physics this semester. I have created this resource document to help you review some of the topics you have been introduced to this semester to better prepare for your Final in physics.

Keywords: Simple Harmonic Motion, Wave Motion, Pendulums, Energy in Simple Harmonic Motion

Important Notes
Important Conventions

Simple Harmonic Motion:



Objects that oscillate over the same path in which the oscillation takes the same amount of time is a periodic motion. Objects in simple harmonic motion are in periodic motion with a restoring force pulling back the object to the equilibrium position when the object reaches its maximum displacement. The restoring force is directly proportional to the displacement of the object. The best example that represents simple harmonic motion is and object attached to a spring. The restoring force that pulls on the object is from the spring. When the spring is compressed or stretched, it exerts a force according to

$$F = -kx.$$

The spring has an external force exerted on it when it is stretched or compressed that acts in the opposite direction.

Due to the periodic motion of the object in harmonic motion, there are a few components of the motion that are very important to understand. Amplitude is the maximum displacement from the equilibrium point. Period (T) is the time it takes for one oscillation. Frequency (f) is the number of oscillations that occur in a second. The relation between frequency and period is as follows.

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$

Energy in Simple Harmonic Motion

Due to the presence of a spring in the system, the mechanical energy of the system is the sum of the kinetic energy of the object and the potential spring energy of its displacement. Since the object in harmonic motion has an amplitude, its velocity at the amplitude is 0. So the mechanical energy is also equal to the potential spring energy at the amplitude of the oscillation.

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2.$$

From this equation, the velocity can be derived as a function of the position. We can find the maximum possible and the velocity at a certain position using the following.

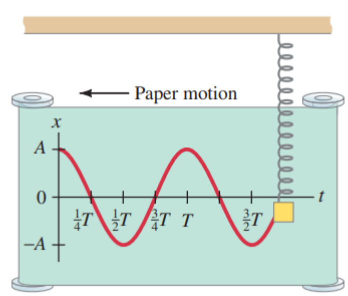
$$v_{max} = \sqrt{\frac{k}{m}}A \qquad v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

Sinusoidal Motion:

The mass and the spring constant also affect the period and frequency of oscillation of a simple harmonic oscillator.

$$T = 2\pi\sqrt{\frac{m}{k}}. \qquad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

The motion of a simple harmonic oscillator can also be see graphically. The motion forms a sinusoidal curve. The sinusoidal nature of the curve allows us to use functions to analyze the motion of the objects. We can use the following functions to understand the motion of the object in terms of position, velocity, and acceleration. What is most important about this section is to understand which variable means what and how it affects the behavior of the wave.



$$x = A \sin \omega t = A \sin(2\pi t/T)$$

$$v = -v_{max} \sin \omega t = -v_{max} \sin(2\pi f t) = -v_{max} \sin(2\pi t/T).$$

$$v_{max} = 2\pi A f = A\sqrt{\frac{k}{m}}.$$

$$a = \frac{F}{m} = \frac{-kx}{m} = -\left(\frac{kA}{m}\right)\cos \omega t = -a_{max} \cos(2\pi t/T)$$

$$a_{max} = kA/m.$$

Example:

The displacement of an object is described by the following equation, where x is in meters and is in seconds:

$$x = (0.30\,\text{m}) \cos(8.0\,t).$$

Determine the oscillating object's amplitude, frequency, period, maximum speed, and maximum acceleration

Solution

From the given function

Amplitude = 0.3 m $2\pi ft = 8$, for t = 1 s $f = 8/2\pi$ f (frequency) = 1.27 Hz

$T = 1/f = 1/1.27 = 0.79$ s

$v_{max} = 2\pi ft = 2\pi\,(1.27)\,(0.3) = 2.4$ m/s

$a_{max} = Ak/m = (0.3)(2\pi f)^2 = (0.3)(2\pi\,(1.27))^2 = 19\ \text{m/s}^2$

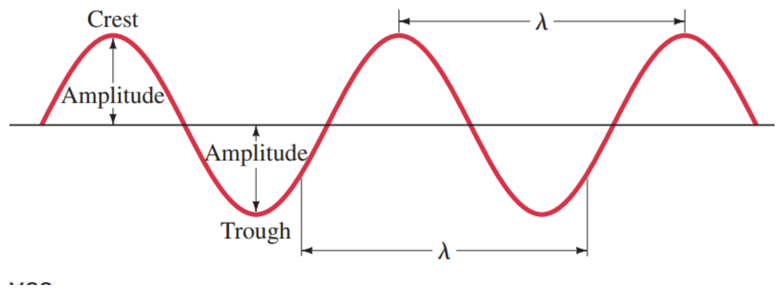
Simple Pendulum

Another system that shows simple harmonic motion is the simple pendulum. Much like the spring system, the simple pendulum is governed by restoring force and a periodic motion. The pendulum is governed by gravity. So, the equations that govern its motion change to the follows.

$$F = -mg \sin \theta, \qquad T = 2\pi\sqrt{\frac{\ell}{g}}. \qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}.$$

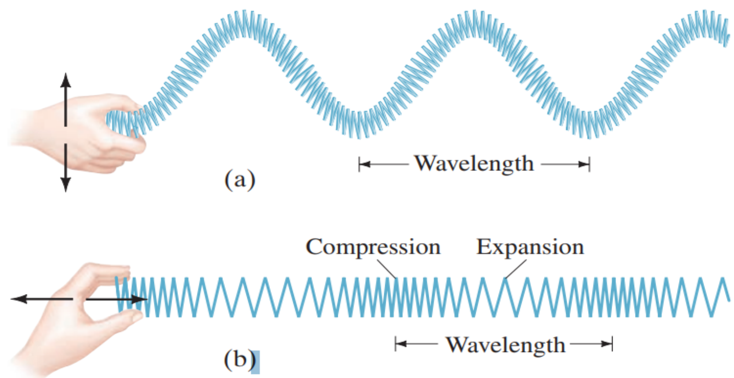
Wave Motion

Mechanical waves are waves the propagate as oscillations in a medium. Waves carry energy. A wave has many different components as shown in the figure. Waves also have a wave speed for each of the crests. The wave speeds can be calculated using the following equation.



$$v = f\lambda$$

There two types of wave motion. A transverse wave motion and a longitudinal wave motion. They look like the following figure. The (a) wave represents a longitunal wave and wave (b) represents a transverse wave.



What is most important to learn from this section is the behavior of waves and functions that govern their oscillation behavior. This section primarily involves the equations mentioned and using the right ones in the problems. Focus on using the practice problems to learn which ones to utilize in what problems. This section does not require major conceptual understanding

All Images are from Physics: Principles with Applications (7th Edition) by Douglas C. Giancoli