**Physics 1408/1420**

**Aman Patel**

Hello Fellow Physicists

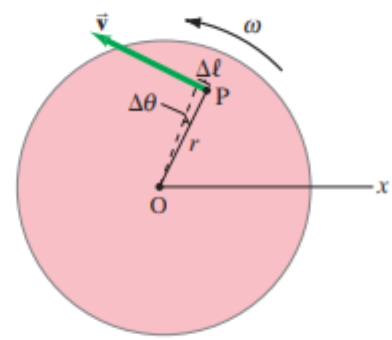
I am Aman Patel, the Master Tutor for Physics this semester. I have created this resource document to help you review some of the topics you have been introduced to this semester to better prepare for your Final in physics.

**Keywords:** Rotational Motion, Angular Kinematics, Torque, Angular Momentum

Important Notes

Important Conventions

## Angular Kinematics:




Angular kinematics analyzes rotational motion. the distinction between rotational motion and circular motion is very important. The motion of earth around the sun is circular motion. The movement of earth on its axis is rotation. Much like how we can analyze linear motion, we can use kinematic variables and equations with rotation. Our variables do change. When looking at rotation, change of angle is displacement, velocity is equivalent to angular velocity, and acceleration is equivalent to angular acceleration. The easiest way to understand the motions is to compare them to one another. The equations all work the same way but only in different scenarios.

| Angular | Linear |
|---|---|
| $\omega = \omega_0 + \alpha t$ | $v = v_0 + at$ |
| $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $x = v_0 t + \frac{1}{2}at^2$ |
| $\omega^2 = \omega_0^2 + 2\alpha\theta$ | $v^2 = v_0^2 + 2ax$ |

| | | |
|---|---|---|
| $x$ | displacement | $\theta$ |
| $v$ | velocity | $\omega$ |
| $a_{tan}$ | acceleration | $\alpha$ |

$$\omega = 2\pi f.$$

$$T = \frac{1}{f}.$$

$$1\ \text{Hz} = 1\ \text{rev/s}.$$

$$x = r\theta$$

$$v = r\omega$$

$$a_{tan} = r\alpha$$

The rotational variables can also relate to tangential linear motion variables. These are also shown above. Let’s look at an example problem.

**Example**

A centrifuge rotor is accelerated for 30 s from rest to 20,000 rpm (revolutions per minute).

(a) What is its average angular acceleration?
(b) Through how many revolutions has the centrifuge rotor turned during its acceleration period?

**Solution**

**(a)**

$\omega_0 = 0$ rad / s

$\omega = 2\pi f$

$= 2\pi\ (20{,}000 / 60)$

$= 2100$ rad / s

$\alpha = (\omega - \omega_0) / \Delta t$

$= (2100 - 0) / 30$

$= 70$ rad / s$^2$

(b)

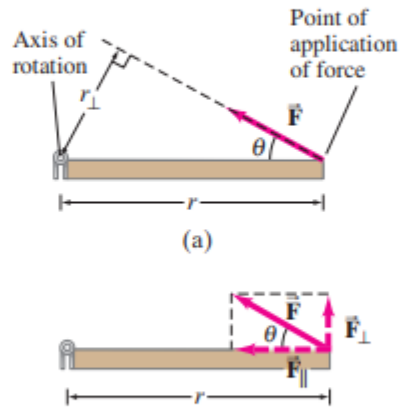
$\Theta = \omega_0 t + (1/2)\alpha t^2$

$= 0(30) + (1/2)(70)(30)^2$

$= 3150$ rad

## **Torque:**

Torque is the equivalent of force in terms of rotation. One thing I will point out is that they are equivalent but no the same. This description is so that you can better visualize these variables and use concepts you are already familiar with to understand this new concept. Torque applies to rotate an object. Every single one of us experiences and applies torque. Have you ever wondered why we put doorknobs at the opposite perimeter of the bracket that attaches the door to the wall? That is because that exerts the most amount of torque. The torque exerted is the product of the perpendicular force and the distance from the axis of rotation. This can vary as it can also be the perpendicular distance from the axis of rotation. Therefore, torque can be calculated using the following




$$\tau = rF_\perp. \qquad \tau = r_\perp F, \qquad \tau = rF\sin\theta$$

## **Moment of Inertia:**

Now we come to what we can think of a mass for a rotating object. Momentum of inertia is the rotational inertia of a rotating object. Generally, moment of inertia is represented by $mr^2$, but different objects have different moments of inertia. A fun experiment you can do to understand how moment of inertia affects rotation, get in a rollie chair and start spinning. First extend your arms and legs, then bring them closer together. You will see that you spin faster. This is because your moment of inertia increases. Moment of Inertia is related to torque as well. If you relate the force equation and the torque equation, you get

$$\Sigma\tau = (\Sigma mr^2)\alpha \qquad \Sigma\tau = I\alpha.$$

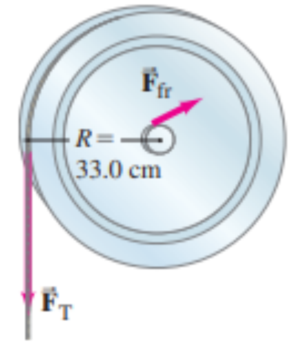
## **Angular Momentum:**

This is the analog of linear momentum with rotation. Angular momentum also follows the law of conservation. If the total angular momentum of a rotating object remains constant if the net torque acting on it is zero.

$$L = I\omega, \qquad \Sigma\tau = \frac{\Delta L}{\Delta t},$$

**Example:**

A 15 N force is applied to a cord wrapped around a pully of mass = 4 kg and radius R = 33 cm. the pully accelerates uniformly for rest to an angular speed of 30 rad / s in 3 s. If there is a frictional torque of 1.1 m.N at the axle, determine the moment of inertia of the pulley.




**Solution**

$\sum \tau = \tau_{tension} - \tau_{pully}$

$= F_{tension}\, R - \tau_{pully}$

$= (0.33)(15) - (1.1)$

$= 3.85$ m.N

$\alpha = (\Delta\omega / \Delta t)$

$= 30 / 3$

$= 10$ rad / $s^2$

$I = ( \sum\tau / \alpha )$

$= 3.85 / 10$

$= 0.385$ kg . $m^2$

All images are from Physics: Principles with Applications ($7^{th}$ Edition) by Douglas C. Giancoli