Physics 1408/1420

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Hello Fellow Physicists

I am Aman Patel, the Master Tutor for Physics this semester. I have created this resource document to help you review some of the topics you have been introduced to this semester to better prepare for your Final in physics.

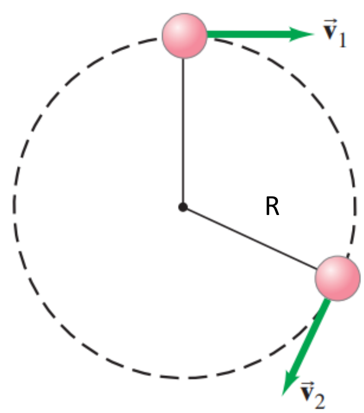
Keywords: Uniform Circular Motion, Centripetal Force, Universal Law of Gravitation, Satellites

Important Notes

Important Conventions

Uniform Circular Motion

An object is said to be uniform circular motion when the object is moving in a circular path at constant speed. If you have ever seen movies, videos and cartoons about cowboys in the old west, you've seen the classic lasso move where they move it over their head, that is uniform circular motion (we consider no air resistance in these scenarios). If you have ever been on a Ferris wheel, you've been an object in circular motion. in uniform circular motion, the distinction between speed and velocity is paramount. In uniform circular motion, the SPEED is CONSTANT but NOT the VELOCITY. When an object is in Uniform circular motion, the magnitude of the velocity does not change, but the direction does. An object can participate in uniform circular motion if there is a centripetal force acting on the object. This is a force that exerts acceleration on the moving object toward the center of the circular path. This acceleration changes the direction of the velocity, which is tangential to the path, as you can see in the figure on the right. This is very important to remember for conceptual questions. This acceleration is called centripetal or radial acceleration, which is defined by the formula below



$$a_c = v^2/ R \qquad \text{(centripetal acceleration)}$$

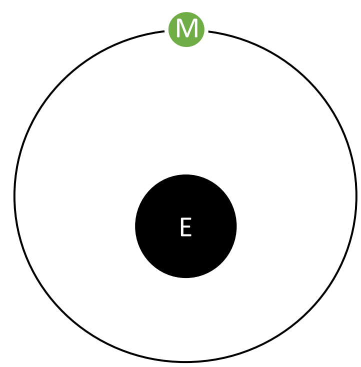
This formula determines the magnitude of the objects acceleration. From this and the F = ma relation of force, we can define the formula of centripetal force as

$$F_c = mv^2/ R$$

Two other terms you must also be aware of are Period (T) and Frequency (f). Period (T) is the time it takes the object in motion to complete one cycle or revolution. Frequency (f) is the amount of cycles completed in a second. Their relation to each other and velocity for uniform circular motion are as follows.

$$f = 1/T \qquad v = 2\pi Rf = 2\pi R/T$$

Example 1: The Moon revolves around the Earth in a circular orbit of radius 384,000 km and a period of 27.3 days. What is the acceleration of the Moon toward the Earth.



Solution:

Radius = 384000 km = 384000000 m

Period = 27.3 days = 27.3x24x60x60 = 2360000 s

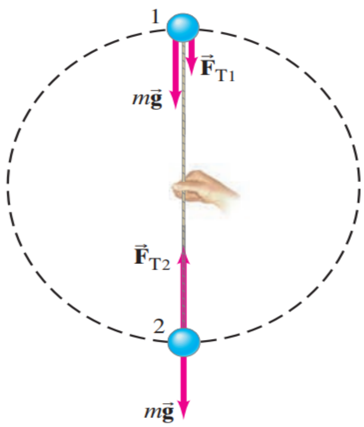
$a_c = v^2/R$

$= (2\pi R)^2/RT^2$

$= (2\pi(384000000))^2/ (384000000)(2360000)$

$= 0.0027\ m/s^2$

Example 2: a 0.150-kg ball on the end of a 1.1 m long cord is swung in a vertical circle. Determine the minimum speed the ball must have at the top of its arc so that it can continue its circular motion and calculate the tension in the cord at the bottom of the path, assume the ball is moving at twice the speed at the top.



Solution:

$m = 0.15$ kg $R = 1.1$ m

At 1

$\sum F = m a_c$

$F_{t1} + F_g = m (v^2/R)$

At the top, for minimum speed F_{t1} will be 0.

$mg = m (v^2/R)$

$9.8 = v^2 / 1.1$ $v = 3.28$ m/s

At 2

$\sum F = m a_c$

$F_{t2} + F_g = m ((2v)^2/R$

$F_{t2} = (0.15) ((2x3.28)^2/ (1.1)) – (0.15) (9.8)$

$F_{t2} = 7.35$ N

Gravity

If you are a fan of Star Wars, guess what, you have the force in you. We call this force gravity. All matter in existence exerts a gravitational force on all other matter. Einstein describes it as a sort of depression in space time. Theoretically, you make a weird ditch in the fabric of space and time, which is what attracts all matter to you. The bigger the object, the more gravitational force it exerts. We don't magically fly to one another because earth has a stronger and deeper spacetime ditch. Newton studied the behavior of gravity not only on Earth, but also out there in space between planets. He proposed the law of universal gravitation, which states: every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. From his statement, we derive the formula for gravity as

$$F_G = G\, m_1\, m_2 / R^2 \qquad \text{where } G = 6.67 \text{ x } 10^{-11} \text{ N.m}^2/\text{kg}^2$$

The relation between the force of gravity and the distance between the particles is called the inverse square law. It is essential to understand how the changes in distance between particles affects the gravitational force between the particles. The relation above changes the closer we get to the surface of a planet due to this law. For earth, you can equate

$$m\, g = (GMm)/(R_{earth})^2 \quad \text{where M = mass of earth, m = mass of object}$$

Another relation that is important is the relationship between gravity and centripetal force. This relation is what creates orbits. Centripetal acceleration occurs when a force is pulling the object in motion toward the center of the circular path. For satellites, it is gravity. We use the following when dealing with such a situation

$$G\, m_{satellite}\, m_{planet} / R^2 = m_{satellite}\, v^2 / R$$

Example 3: A geosynchronous satellite is one that stay above the same point on the earth on the equator. What is the height of the satellite above the Earth and the speed of the satellite?

Solution:

$G\, m_{satellite}\, m_{planet} / R^2 = m_{satellite}\, v^2 / R$

$G\, m_{planet} / R^2 = (2\pi R/T)^2 / R$

$G\, T^2\, m_{planet} / 4\pi^2 = R^3$

$R = (\, 6.67 \times 10^{-11} \times (\, 24 \times 60 \times 60\,)^2\, (5.98 \times 10^{24}) / 4\pi^2\,)^{1/3}$

$= 4.22 \times 10^7$ m

Remember, this is the distance of the satellite from the center of the earth, it is NOT you answer. You want the height. So,

Height of Satellite = $R - R_{earth} = 4.22\text{x}10^7 - 6.38\text{x}10^6 = 3.58\text{x}10^7$ m

For velocity of satellite

$v = 2\pi R/T$

$= 2\pi(4.22 \times 10^7)/\,(24\text{x}60\text{x}60)$

= 3070 m/s

It is fascinating how these laws govern everything we see out there. The next time you look at the sky, take a moment to think about how everything you have learned applies to everything out there and how the laws of physics make up the most beautiful masterpiece we call the universe.

All images are from Physics: Principles with Applications (7th Edition) by Douglas C. Giancoli