Calculus 1, Week 11

Hey Calculus tutors and students! This resource covers the eleventh week of class. Specifically, in this resource I will cover Riemann Sums, Definite Integrals, and Indefinite Integrals.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

Keywords: Riemann Sums, Definite Integrals, Indefinite Integrals.

Key

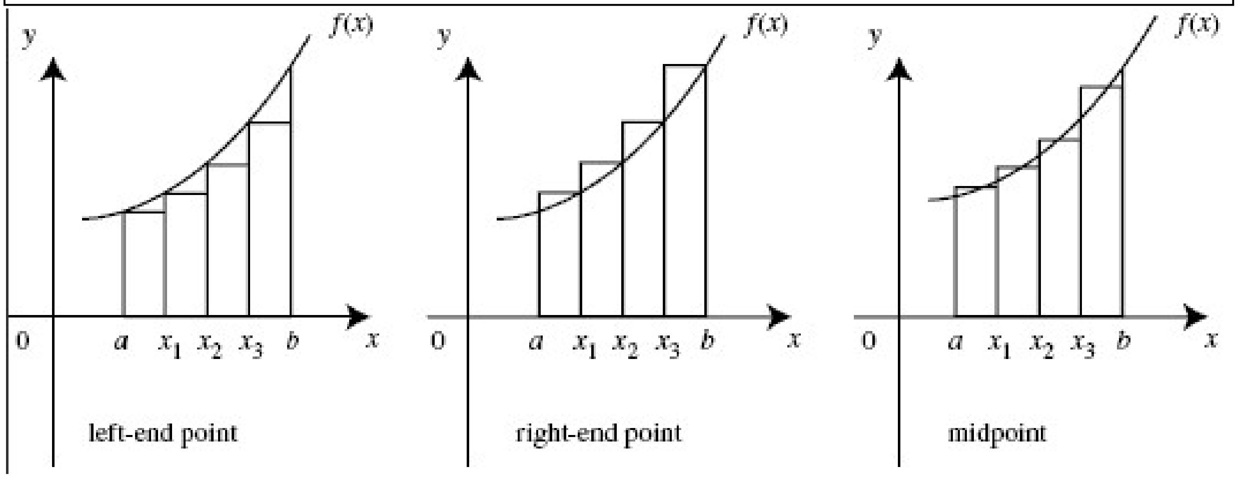
- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

Chapter 5.1

- In chapter 5.1 the we will be learning about how to approximate the area under a curve using right, left, and middle Reimann Sums. A Riemann Sum is an approximation of the area underneath a curve through the use of rectangles. These rectangles take on the height of the graph at some point on the interval of the rectangle – usually the right, left, or middle of said rectangle. By summing up the areas of all the rectangles you created, you can find an approximant area of the area under this curve.
- Video Resource
  - https://www.youtube.com/watch?v=gK0K1XptyNA
- Example Problem
  - What is the Riemann approximation of the area under $f(x) = x^2$ on the interval [1,4] using a left-hand approximation and three rectangles of equal Δx? Is this an over or under estimation of the actual area under the curve?
    - A: $L_3 = (1 * 1) + (4 * 1) + (9 * 1) = 14.$
    - This is an underestimate of the area.

The Three kinds of Riemann Sums. Source: Reflections of a Second-career Math Teacher On WordPress https://mathequality.wordpress.com/2013/01/09/building-a-riemann-sum-spreadsheet/



- What is the Riemann approximation of the area under $f(x) = 3x^2 - 5x + 7$ on the interval [0,6] using a right-hand approximation and three rectangles of equal Δx? Is this an over or under estimation of the actual area under the curve?
  - A: $R_3 = (9 * 2) + (35 * 2) + (85 * 2) = 258.$
  - Technically, since this function is not monotonic, we don't know if this is an over or under estimation.
- What is the Riemann approximation of the area under $f(x) = ln(x)$ on the interval [0,6] using a midpoint approximation and three rectangles of equal Δx? Is this an over or under estimation of the actual area under the curve?
  - A: $R_3 = (9 * 2) + (35 * 2) + (85 * 2) = 258.$
  - Since we are using a midpoint approximation, we don't know if this is an over or under estimation.

Chapter 5.2

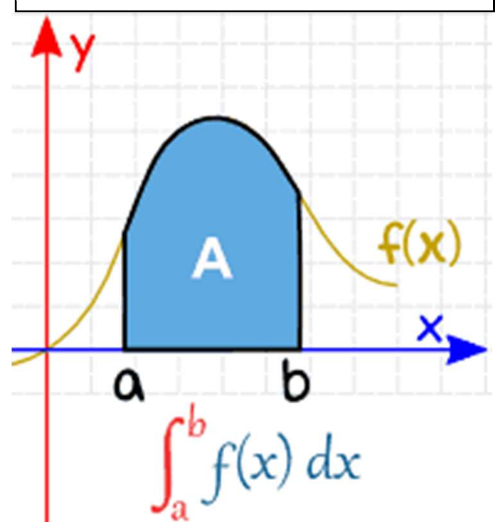
- In chapter 5.2 we will be learning about the definite integral (which just means an integral over a specific interval). Theoretically, we get an integral by taking Reimann Sum over a specific interval with more and more rectangles. As the number of rectangles goes to infinity, each rectangle becomes infinitely thin. In practice, we can use a variety of shortcuts we can use when taking integrals, seen below.
  - $\int_a^b c dx = c(b - a)$
  - $\int_a^b x^n dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$
  - $\int_a^b f(x)dx = -\int_b^a f(x)dx$
  - $\int_a^a f(x)dx = 0$

- $\int_a^b cf(x)dx = c\int_a^b f(x)dx$
- $\int_a^b (\mathrm{f(x)} + \mathrm{g(x)})dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
- $If\ m \leq f(x) \leq M\ on\ the\ interval\ (a, b), then\ \int_a^b mdx \leq \int_a^b f(x)dx \leq \int_a^b Mdx$
    - This theorem also works with functions in place of m and M, and also works with only one bound.

- Video Resource
    - https://www.youtube.com/watch?v=D9o6Gbg77OM
- Example Problem
    - $\int_2^4 x^2 dx$
        - $\frac{(4)^3-(2)^3}{3}$
    - $\int_{-1}^7 x^4 dx$
        - $\frac{(7)^5-(-1)^5}{5}$
    - $\int_5^3 x^7 dx$
        - $-\frac{(5)^8-(3)^8}{8}$
    - Find upper and lower bounds for $\int_0^3 \sin(x)dx$
        - $\int_0^3 (-1)dx \leq \int_0^3 \sin(x)dx \leq \int_0^3 (1)dx$

$$-3 \leq \int_0^3 \sin(x)dx \leq 3$$

A Graphic Representation of a Definite Integral

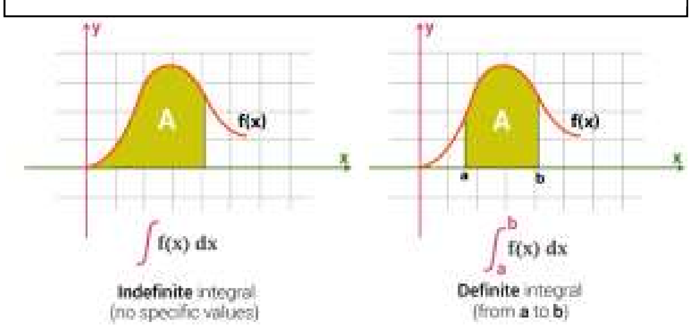


Source: Math Is Fun https://www.mathsisfun.com/calculus/integration-definite.html

Chapter 5.3

- In chapter 5.3 we first learn that integrals are considered the opposite of a derivative. The rules of integration introduced in chapters 5.2 and 5.3 should look familiar – they are dust the revers of the rules of derivatives already introduced.
- After that we learn about indefinite integrals. The only difference between calculating definite and indefinite integrals is the presence (or lack thereof) of endpoints of the integral. Since the indefinite integral doesn't have bounds, two things will be different from calculating a definite integral. 1st, we will get an equation for our answer, rather than a value. 2nd, we need to add a "+ C" to all of our

A Definite Integral is along some interval A to B, while an Indefinite Integral lacks specific endpoints



Source: https://byjus.com/maths/indefinte-integrals/

equations. Also in this section we are introduced to some new integrals.

- $\int 0dx = c$
- $\int \sin(kx)dx = -\frac{1}{k}\cos(kx) + c$
- $\int \cos(kx)dx = \frac{1}{k}\sin(kx) + c$
- $\int \mathrm{e}^{\mathrm{kx}}dx = \frac{1}{k}e^{kx} + c$
- $\int \frac{1}{\mathrm{x}}dx = ln|x| + c$

- When taking an integral, you should always be able to take the derivative of your answer and get what you originally started with. Thus, it is possible to use derivatives to check to make sure you got the answer correct.
- Video Resource
  - https://www.youtube.com/watch?v=6yXSp3FPkqY
- Example Problem
  - $f(x) = \int (x^2 + \frac{1}{x})dx$
    - $f(x) = \frac{x^3}{3} + ln|x| + c$
    - To check our work we can calculate $f'(x)$
      - $f'(x) = 3\frac{x^2}{3} + \frac{1}{x} + 0$
      - This is exactly what we started with!
  - $f(x) = \int (\sin(5x) + cos(3x)dx$
    - $f(x) = -\frac{1}{5}\cos(5x) + \frac{1}{3}\sin(3x) + c$
    - To check our work we can calculate $f'(x)$
    - $f'(x) = (-\frac{1}{5}) * 5 * \cos(5x) + (\frac{1}{3}) * 3 * \sin(3x) + 0$
  - $f(x) = \int (\mathrm{e}^{7\mathrm{x}} + \frac{1}{\mathrm{e}^{\mathrm{x}}})dx$
    - $f(x) = \frac{1}{7}e^{7x} + \frac{1}{-1e^x} + c$
    - To check our work we can calculate $f'(x)$
      - $f'(x) = (\frac{1}{7}) * 7 * e^{7x} + (-1) * \frac{1}{-1e^x} + 0$

Things Students Tend to Struggle With

- **Remembering the difference between the 3 kinds of Reimann sums.**
  - Remembering what a midpoint Reimann Sum looks like is usually pretty easy, but some students can get Left and Right End Reimann Sums mixed up. Once you know where to start the rectangles the actual calculation of the Reimann Sum is less calculus and more geometry.
- **Remembering to put "+ C" at the end of indefinite integrals.**
  - It is a really easy thing to forget, but whenever you are calculating an indefinite integral (the ones without the bounds) you need to be sure to include a "+ C" at the end. It's an easy mistake that can cost you quite a few points on a test.