Calculus 1, Week 9

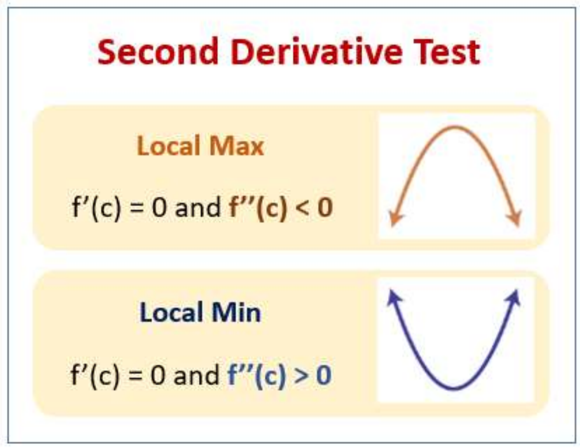
Hey Calculus tutors and students! This resource covers the ninth week of class. Specifically, in this resource I will cover how to make use the 2nd derivative test in order to find local maxes and mins for a function.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

Keywords: Critical Points, 2nd Derivative Test.



Source: https://www.onlinemathlearning.com/second-derivative.html

Key

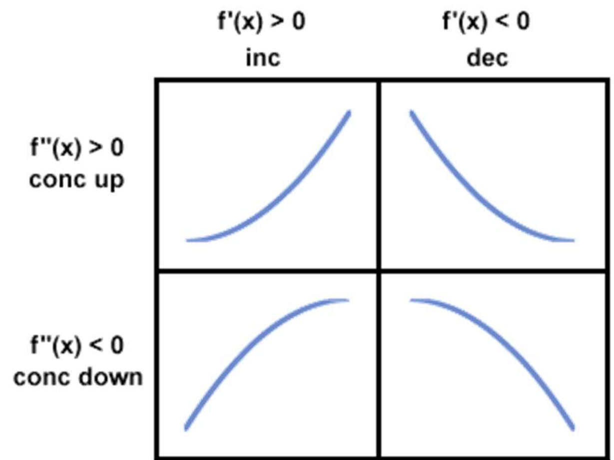
- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

Chapter 4.4

- In chapter 4.4 the students will be learning about the second derivative test and how to identify a graph based on its first and second derivatives. The key take away from this section is that the second derivative of a graph determines where the graph is concave up and where it is concave down. We can use the concavity at a critical point to determine if a critical point is a local max or min, as shown in the graphic above. Finally, an inflection point is where the concavity of a graph equals zero or does not exist. Between inflection points concavity doesn't switch between positive and negative – just like with the first derivative between critical points.
- Video Resource
  - https://www.youtube.com/watch?v=l1YHElEgbog&feature=youtu.be

- Example Problem



Source: SparkNotes

https://www.sparknotes.com/math/calcab/applicationsofthederivati

  - Where is the graph of $f(x) = x^3 - 3x$ concave up and concave down?
    - It is concave down from negative infinity to 0 and concave up from 0 to infinity.
  - Determine the Critical Points of $f(x) = x^3 - 3x$ and use the 2nd derivative test to determine if they are local maxes or local mins.
    - Critical Points: $x = -1\ (max)$ and $x = 1\ (min)$
  - Where is the graph of $f(x) = x^3 + 3x^2 - 9x$ concave up and concave down?
    - It is concave down from negative infinity to -1 and concave up from -1 to infinity.
  - Determine the Critical Points of $f(x) = x^3 + 3x^2 - 9x$ and use the 2nd derivative test to determine if they are local maxes or local mins.
    - Critical Points: $x = -3\ (max)$ and $x = 1\ (min)$
  - Where is the graph of $f(x) = 2x^3 + 3x^2 - 36x$ concave up and concave down?
    - It is concave down from negative infinity to $-\frac{1}{2}$ and concave up from $-\frac{1}{2}$ to infinity.
  - Determine the Critical Points of $f(x) = 2x^3 + 3x^2 - 36x$ and use the 2nd derivative test to determine if they are local maxes or local mins.
    - Critical Points: $x = -3\ (max)$ and $x = 2\ (min)$
  - Where is the graph of $f(x) = x^5 - 5x^4$ concave up and concave down?
    - It is concave down from negative infinity to 3, and concave up from 3 to positive infinity.
  - Determine the Critical Points of $f(x) = x^5 - 5x^4$ and use the 2nd derivative test to determine if they are local maxes or local mins.
    - Critical Points: $x = 0\ (max)$ and $x = 4\ (min)$
  - Where is the graph of $f(x) = \frac{-x^2}{x^2+1}$ concave up and concave down?
    - It is concave up from negative infinity to $\frac{-1}{\sqrt{3}}$, concave down from $\frac{-1}{\sqrt{3}}$ to $\frac{1}{\sqrt{3}}$, and concave up from $\frac{1}{\sqrt{3}}$ to infinity.
  - Determine the Critical Points of $f(x) = \frac{-x^2}{x^2+1}$ and use the 2nd derivative test to determine if they are local maxes or local mins.
    - Critical Points: $x = 0\ (max)$

- Where is the graph of $f(x) = x^4e^x$ concave up and concave down?
    - It is concave up from negative infinity to -6, concave down from -6 to -2, concave up from -2 to 0, and then concave up again from 0 to infinity.
        - Despite the fact that the function is concave up from [-2,0) and (0, inf), at zero the function has a concavity of zero.
- Determine the Critical Points of $f(x) = x^4e^x$ and use the 2nd derivative test to determine if they are local maxes or local mins.
    - Critical Points: $x = -4\ (max)$ and $x = 0\ (unknown)$
        - Follow Up Question: Why can we not use the 2nd derivative test to determine whether $x = 0$ is a local max or min? How could we figure this out?
            - A: We can't use the 2nd derivative test in this situation because at $x = 0$ the 2nd derivative equals zero. As such, the 2nd derivative test fails. If we use the 1st derivative test though, we can determine that this is a local minimum.

Things Students Tend to Struggle With

- **2nd Derivative Test**
    - The 2nd Derivative Test is another powerful tool for identifying local extrema and sketching what a function looks like on a graph. In many ways using the 2nd derivative test is similar to using the 1st derivative test. As with the 1st derivative test I like to draw out the number line for x and then separating the line into different segments. However, when using the 2nd derivative test I break the number line into segments based on Inflection points (where the 2nd derivative equals zero) rather than places where the 1st derivative equals zero. Once I determine what the concavity is for each segment of my number line we should be able to determine whether each critical point is a local max or min. Importantly, the 2nd derivative is incapable of determining the location of saddle points.