Calculus 1, Week 8

Hey Calculus tutors and students! This resource covers the sixth week of class. Specifically, in this resource I will cover how to make a linear approximation of a function, what a critical point is, what the Mean Value Theorem and Monotonicity say, and how we can use them to develop the 1st derivative test.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson

Keywords: Linear Approximation, Critical Points, Mean Value Theorem, Monotonicity, 1st derivative test.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

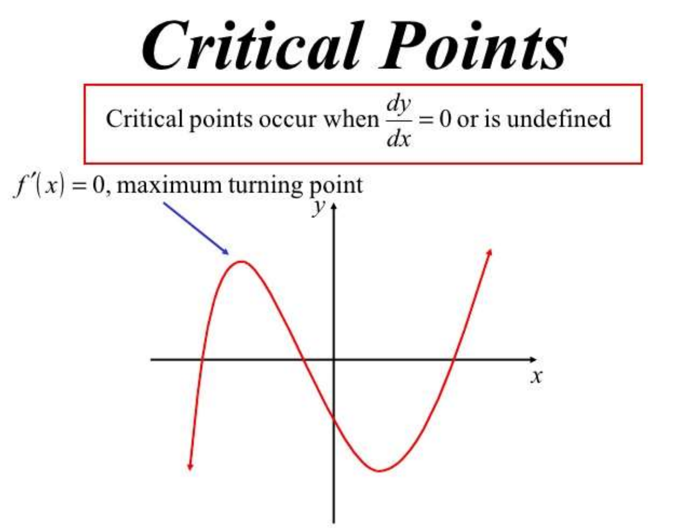
Chapter 4.1

- In this chapter the book introduces linear approximations of functions. The linear approximation of a function is defined as such
  - $L(x) = f(a) + f'(a)(x - a)$, with $L(x) \approx f(x)$
  - In differential notation we treat $\Delta y \approx dy$ and $\Delta x \approx dx$ for a small dy and dx.
  - The error and percentage error for of our Linear Approximation is defined as
    - $error = |\Delta f - f'(a)\Delta x|$
    - $\%\ error = \left|\frac{error}{actual\ value}\right| * 100\%$
- Example Problem
  - Estimate x^3 from the point x=2 to the point x=2.03. What is the % error of your estimation?
    - $L(2.03) = 8 + (12)(0.03) = 8.36$

        - $\% \, error = \left|\frac{8.3654-8.36}{8.3654}\right| * 100\% = \; .06\%$
    - Estimate $\sqrt[3]{x}$ from the point x=8 to the point x=8.12. What is the % error of your estimation?
        - $L(8.12) = 2 + (1/12)(0.12) = 2.01$
        - $\% \, error = \left|\frac{2.00995 \quad .01}{2.00996}\right| = \; .003\%$

Chapter 4.2

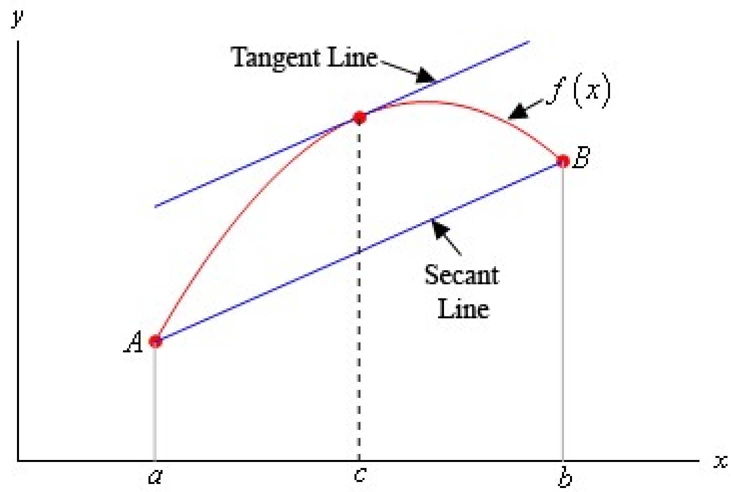
- In this chapter the classes are going over finding local maximums and minimums using Calculus by finding critical points, then observing how the function looks at either side of the point.
- Video Resources
    - https://www.youtube.com/watch?v=nMsn8Txk8to&feature=youtu.be
- Example Problem
    - What is are the critical points of f(x) = $x^2 - 10x + 25$? Are they maximums or minimums? What is the Absolute Max and Absolute Min on the interval [-10, 10]
        - Critical point at x= 5 (local min).
        - Absolute Max at x= - 10. Absolute Min at x=5
    - What are the critical points of $f(x) = x^4 - 4x^3$? Are they maximums or minimums? What is the function's Absolute Max and Min on the interval [-5, 5]?
        - A: Critical points at x = 0 (Saddle Point) and x = 3 (Local Min). Absolute Max at x = - 5, Absolute Min at x = 3
    - What are the critical points of $f(x) = e^x + x^3 - 25x$? Are they maximums or minimums?
        - Critical points are at x= -2.577 (local max) and 2.031 (local min).
    - What is the Absolute Maximum of $f(x) = x^4 - 4$ on the interval [-4, 4]? The Absolute Minimum?
        - Min: x = 0
        - Max: x = 4 or - 4



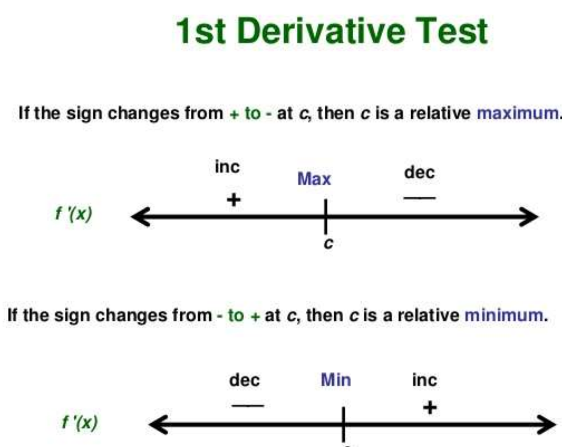
Source: https://www.slideshare.net/nsimmons/11x1-t09-02-critical-points

Chapter 4.3

- In chapter 4.3 the students will be learning the Mean Value Theorem (MVT), the first derivative test, and Monotonicity. The MVT states that, across an interval of a continuous function, at some point between the endpoints of the interval there will be a point "c" such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. Monotonicity states that between two critical points a function will always be increasing or decreasing. From this, we can develop the first derivative test.
- Example Problem
  - Find a "c" satisfying the conclusion of MVT for $f(x) = x^3$ on the interval [0,5].
    - The point x=2.887
  - Find a "c" satisfying the conclusion of MVT for $f(x) = \frac{1}{x}$ on the interval [1,5].
    - The point $x = \sqrt{5}$
  - Find the critical points of the function $f(x) = x^3 - 3x^2$ and determine if they are local maximums or minimums using the first derivative test.
    - The two points are x=0 (a local max) and x=2 (a local min)
  - Find the critical points of the function $f(x) = \frac{-x}{x^2+1}$ and determine if they are local maximums or minimums using the first derivative test.
    - The two points are x = -1 (a local max) and x = 1 (a local min)
  - Find the critical points of the function $f(x) = e^x x^4$ and determine if they are local maximums or minimums using the first derivative test.
    - The two points are x = -4 (a local max) and x = 0 (a local min)



Source: https://tutorial.math.lamar.edu/classes/calci/MeanValueTheorem.aspx



Source: https://www.slideshare.net/bigkelt12/lesson-33-29000437

Things Students Tend to Struggle With

- **Formula for Linear Approximation**
  - While the formula for the Linear Approximation looks really ugly, it is fairly easy to remember if you can relate it in your mind back to more simple formulas you have learned in the past. If we rewrite the equation of the Linear Approximation from $L(x) = f(a) + f'(a)(x - a)$ to $L(x) - f(a) = f(a) + f'(a)(x - a)$ we can see that this is the same as the point-slope formula of a line from back in geometry. In this case the point is the value of x (which we call a) that is easy to plug into f(x), and the slope of our approximation is the slope of the line at the point x = a (which we write as $f'(a)$).
- **1^{st} Derivative Test**
  - The 1^{st} Derivative Test is an incredibly powerful tool for identifying local extrema and sketching what a function looks like on a graph. In general I find it easiest to use the 1^{st} Derivative Test by drawing out the number line for x and then separating the line into different segments using the points where the slope is equal to zero (as shown in one of the graphics above). From there we should be able to determine whether each critical point is a local max, min, or saddle point.