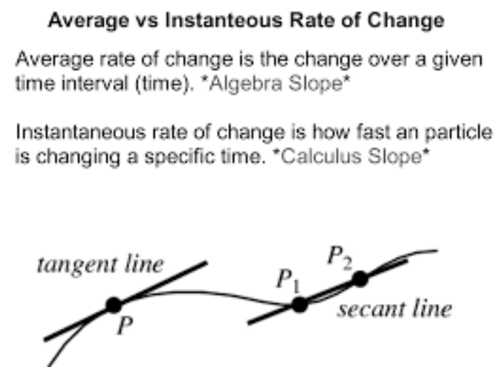
Calculus 1, Week 5

Hey Calculus tutors and students! This resource covers the fifth week of class. Specifically, in this resource I will cover how to work with velocity and acceleration with derivatives, higher order derivatives, and derivatives of trig functions.

In addition to this resource and our one-on-one tutoring sessions, the Baylor Success Center also runs a weekly group tutoring session for Calculus 1. The group tutoring session occurs every Wednesday from 5pm-6pm. If you are interested in attending you can reserve a spot through our website: https://www.baylor.edu/support_programs/index.php?id=40917

As with last year, if anyone has any suggestions for things to add to the resource please do not hesitate to contact me!

- Alex Williamson



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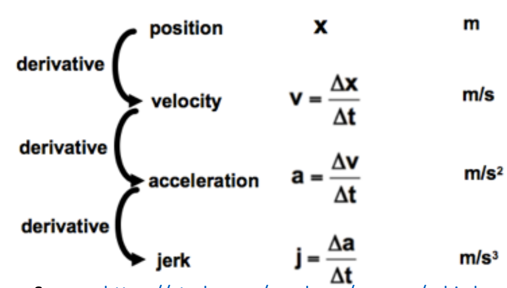
Keywords: Velocity, Acceleration, Higher Order Derivatives, Derivatives of Trig Functions.

Key

- Yellow Highlighting: Definitions that you need to know.
- Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Concepts

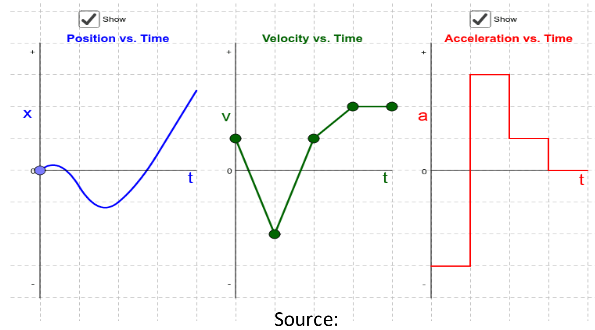
- **Chapter 3.4**
  - In this chapter we go back to the first part of Calc 1 when we were introduced to average vs. instantaneous rates of change, and how to approximate them. Now however, our approximations will make use of differentiation, rather than having to make use of tables.
  - Also in this chapter we learn the relationship between an object's position, velocity, and acceleration. This relationship is shown in my graphic to the right.



Source: https://study.com/academy/answer/a-bird-flies-in-the-xy-plane-with-a-velocity-vector-given-by-velocity-vector-alpha-beta-t-2-i-vector-gamma-t-j-vector-with-alpha-2-4-m-s-beta-1-6-m-s-3-and-gamma-4-0-m-s-2-the-positi.html

  - Video Resources
    - https://www.youtube.com/watch?v=hqtFnWwmjo8&t=3s
  - Example Problem
    - What is an estimate for f(7) if f(8)=4 and f'(8)= -1?

- f(7) ≈ 5
    - Why is the answer not 3?
- **Chapter 3.5**
  - In this section we are introduced to higher order derivatives, like y'', y''', etc. We can describe higher order derivatives as the rate of change of the lower order derivates, as the tangent line of the graph of a lower order derivative, or as the concavity of a graph (although that isn't introduced until chapter 4).



Source: https://www.geogebra.org/m/pdNj3DgD

  - Video Resources
    - https://www.youtube.com/watch?v=lfSNkNIGcQ4
  - Example Problem
    - What is the derivative of f(x) =x^2 ? What is the second derivative?
      - f'(x) = $2x$
      - f''(x) = 2
    - What is the derivative of f(x) =$7x^5 - 21x^3 - 37x - 100$? What is the second derivative?
      - f'(x) = $35x^4 - 63x^2 - 37$
      - f''(x) = $140x^3 - 126x$
- **Chapter 3.6**
  - In this section we are introduced to the derivatives of the standard trig functions. The two most important rules to remember are that $\frac{d}{dx}$sin(x) = cos(x) and that $\frac{d}{dx}\cos(x) = -\sin{(x)}$. some other important rules are
    - $\frac{d}{dx}\tan(x) = sec^2(x)$
    - $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
    - $\frac{d}{dx}\cot(x) = -csc^2(x)$
    - $\frac{d}{dx}\csc(x) = -\csc(x)\cot{(x)}$
  - Video Resources
    - https://www.youtube.com/watch?v=wb2Ru7knz8I
  - Example Problem
    - What is the derivative of f(x) = $\frac{\sin{(x)}}{\cos(x)}$?
      - f'(x) = $sec^2(x)$ or $\frac{\cos(x)*\cos(x)-\sin(x)*(-1)\sin{(x)}}{cos^2(x)}$
        - Follow up, are these equal? (yes, they are)
    - What is the derivative of f(x) = $\sec(x)\cot(x)$?
      - f'(x) = $-\sec(x)\,csc^2(x) + \cot(x)\sec(x)\tan(x)$
    - What is the derivative of f(x) = $\cot(x)$? What is it's second derivative?

        - f'(x) = $-csc^2(x) = -csc(x) * csc(x)$
        - f''(x) = $-csc(x) * (-\csc(x)\cot(x)) + csc(x) * (-1)(-\csc(x)\cot(x))$
    - What is the derivative of f(x) = $\sec(x)\cot(x)$?
        - f'(x) = $-\sec(x)\, csc^2(x) + \cot(x)\sec(x)\tan(x) = -\sec(x)\, csc(x) csc(x) + \sec(x)\frac{tan(x)}{tan(x)}$
        - f''(x) = $-\sec(x)\,((csc(x)) * (-\csc(x)\cot(x)) + (csc(x)) * (-\csc(x)\cot(x))) + csc^2(x) * \sec(x)\tan(x) + \sec(x)\tan(x)$
            - When we learn the Chain Rule in later chapters these sort of problems will get much easier.

<u>Things Students Tend to Struggle With</u>

- **Derivatives of Trig Functions**
    - Many students struggle to remember the derivatives of trig functions – especially trig function that we don't use very often, like sec(x) and csc(x). The key to memorizing derivatives of trig functions is to recognize the patterns. For instance, the derivatives of sec(x) and csc(x) look very similar. The same is true for tan(x) and cot(x).