# MTH 1320 – PRECALCULUS
# FALL 2020 WEEK 14 RESOURCE

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Welcome back, Precalculus scholars! This is the last Precalculus resource for the semester. It focuses on material covered in the fourteenth week of classes, namely trigonometry topics from sections 6.3, 7.2, and 7.3 of OpenStax's *Precalculus*. It also provides a summary of the functions you learned throughout the semester, excluding trigonometric functions. Please refer to previous resources if you would like to review other topics!

## 6.3 Inverse Trigonometric Functions

Recall that the trigonometric functions allow you to convert the measure of an angle to a ratio [1]. There are times when instead we want to find the measure of an angle using a ratio. **Inverse trigonometric functions**, which "undo" the trigonometric functions, allow us to do this. See Figure 1.

| Trig Functions | Inverse Trig Functions |
|---|---|
| Domain: Measure of an angle | Domain: Ratio |
| Range: Ratio | Range: Measure of an angle |

**Figure 1. The Domains and Ranges of Trig and Inverse Trig Functions**
**Source: [1]**

Because the trigonometric functions are not one-to-one, we have to restrict their domains to a portion where they are one-to-one in order to find their inverses. Our choice of the trigonometric functions' restricted domains specifies the inverse trigonometric functions' ranges. By convention, we define the inverse trigonometric functions as shown in Table 1.

**Table 1. The Inverse Trigonometric Functions**

| Inverse Trigonometric Function Name | Notation | Domain | Range |
|---|---|---|---|
| **Inverse Sine or Arcsine Function** | $y = \sin^{-1} x$ or $\arcsin x$ | $[-1, 1]$ | $[-\pi/2, \pi/2]$ |
| **Inverse Cosine or Arccosine Function** | $y = \cos^{-1} x$ or $\arccos x$ | $[-1, 1]$ | $[0, \pi]$ |
| **Inverse Tangent or Arctangent Function** | $y = \tan^{-1} x$ or $\arctan x$ | $(-\infty, \infty)$ | $(-\pi/2, \pi/2)$ |

As with any inverse function, you can graph the inverse trigonometric functions by reflecting the restricted graphs of the trigonometric functions about the line $y = x$ [1]. See Figure 2, Figure 3, and Figure 4 for graphs of the inverse trigonometric functions and the associated trigonometric functions.

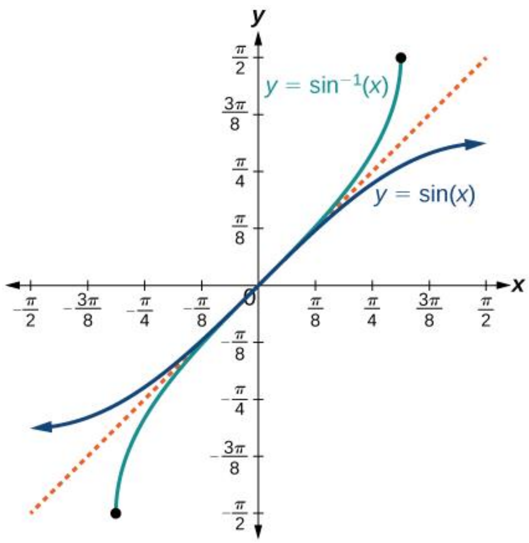


**Figure 2. The Inverse Sine Function**
**Source: [1]**

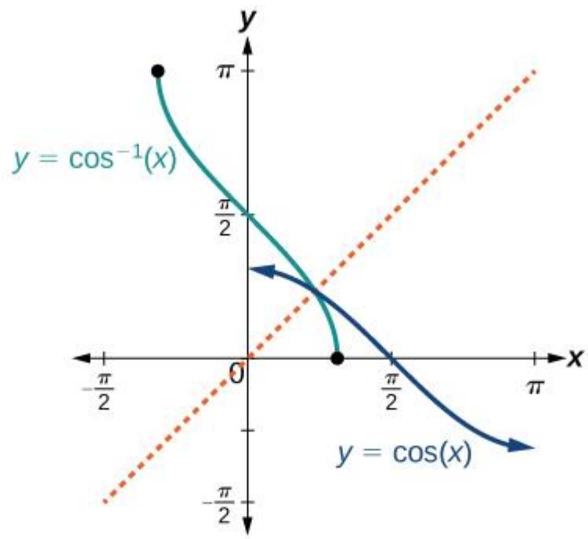


**Figure 3. The Inverse Cosine Function**
**Source: [1]**

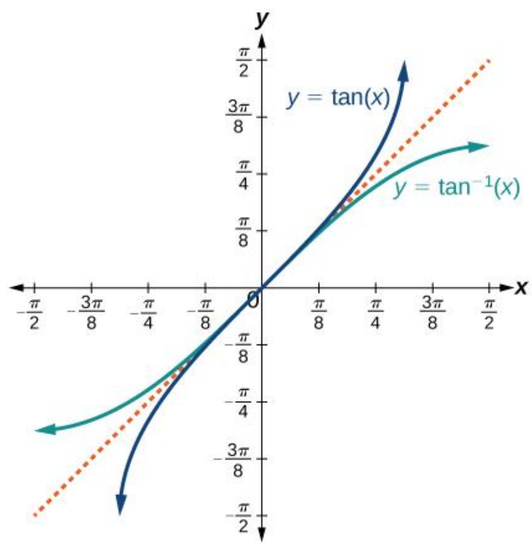


**Figure 4. The Inverse Tangent Function**
**Source: [1]**

To determine the value of an inverse trigonometric function, you can use an approach similar to the approach used for logarithmic functions, which are inverses of exponential functions. For example if you are given an inverse sine function, $y = \sin^{-1} x$, ask yourself, "What angle $y$ results in $\sin y = x$?" The angle is the output of the inverse function, and it must be in the range shown in Table 1.

Recall from section 5.4 that you could use trigonometric functions to find a missing side of a triangle given a side and an angle. Now, we can find an angle of interest using two sides of a triangle with inverse trigonometric functions! The process for this is as follows:

1. Draw and label the write triangle if it is not given.
2. Use Soh Cah Toa to equate a trigonometric function of the angle of interest to a ratio of the two given sides of the triangle.
3. Apply the corresponding inverse trigonometric function to both sides of the equation.

4. Simplify the composition of the inverse and original trigonometric functions. (For example, $\sin^{-1}(\sin x) = x$.)
5. Determine the output of the inverse trigonometric function using your memorized unit circle (for special angles/ratios) or a calculator.

## 7.2 Sum and Difference Identities

Using the **sum and difference identities** for the trigonometric functions, we can evaluate more angle values than just the special angles on the unit circle. If we can rewrite an angle as a sum or difference of two special angles, the corresponding sum or difference formula allows us to expand the given trigonometric function into a combination of trigonometric functions of the two special angles. We know the output values of the trigonometric functions for special angles, so we can find the overall value! See Table 2 for a summary of the sum and difference formulas for cosine, sine, and tangent.

**Table 2. The Sum and Difference Formulas**

| Trigonometric Function | Sum Formula | Difference Formula |
|---|---|---|
| **Cosine** | $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ | $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ |
| **Sine** | $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ | $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ |
| **Tangent** | $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ | $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$ |

We can also the formulas to simplify trigonometric expressions and verify other trigonometric identities.

## 7.3 Double-Angle, Half-Angle, and Reduction Formulas

Section 7.3 covers three more types of formulas for trigonometric functions. We can use the **double-angle formulas** to rewrite a trigonometric function of a double angle $2\theta$ as a combination of trigonometric functions of the angle $\theta$. We can also use the **reduction formulas** "to reduce the power of a given expression involving even powers of sine or cosine" [1]. Lastly, we can use the **half-angle formulas** to rewrite a trigonometric function of a half angle $\theta/2$ with trigonometric functions of $\theta$.

**Table 3. Double-Angle, Half-Angle, and Reduction Formulas**

| Trigonometric Function | Double-Angle Formula | Half-Angle Formula | Reduction Formula |
|---|---|---|---|
| **Cosine** | $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $= 1 - 2\sin^2\theta$ $= 2\cos^2\theta - 1$ | $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$ | $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$ |
| **Sine** | $\sin(2\theta) = 2\sin\theta\cos\theta$ | $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ | $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$ |
| **Tangent** | $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$ | $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ $= \dfrac{\sin\theta}{1+\cos\theta}$ | $\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$ |

| | | | |
|---|---|---|---|
| | | $= \frac{1 - \cos\theta}{\sin\theta}$ | |

## Summary of Functions

This semester, you have learned about many different types of functions. The form and/or notation for these functions is summarized in Table 4 below. Note that the trigonometric functions are not included in the table because there are multiple notations for that type.

**Table 4. Summary of Function Types**

| Type | General Form | Alternate Form |
|---|---|---|
| **Linear** | $f(x) = mx + b$ (slope-intercept form) | $y - y_1 = m(x - x_1)$ (point-slope form) |
| **Quadratic** | $f(x) = ax^2 + bx + c$ | $f(x) = a(x - h)^2 + k$ (standard form) |
| **Power** | $f(x) = kx^p$ | |
| **Polynomial** | $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ | |
| **Rational** | $f(x) = \frac{P(x)}{Q(x)} = \frac{a_p x^p + a_{p-1} x^{p-1} + \cdots + a_2 x^2 + a_1 x + a_0}{b_q x^q + b_{q-1} x^{q-1} + \cdots + b_2 x^2 + b_1 x + b_0}$ | |
| **Exponential** | $f(x) = ab^x$ | |
| **Logarithmic** | $f(x) = \log_b(x)$ | $b^y = x$ |

## Reminders

I hope that the Precalculus resources I shared with you this semester were helpful and efficient in addressing your academic needs. All resources for this semester can be found here: https://www.baylor.edu/support_programs/index.php?id=967950.

Here are a few reminders about Precalculus opportunities:

- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/support_programs/index.php?id=955694.
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at https://www.baylor.edu/tutoring.

## References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: https://openstax.org/details/books/precalculus