MTH 1320 – PRECALCULUS
FALL 2020 WEEK 13 RESOURCE

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Welcome back, Precalculus scholars! We are so close to the end of the semester! This resource focuses on material covered in the thirteenth week of classes, namely graphs of trigonometric functions from sections 6.1-6.2 of OpenStax's *Precalculus*. Please refer to previous resources if you would like to review other topics. Don't forget to sign up for the last session of Group Tutoring on Tuesday, November 17th!

6.1 Graphs of the Sine and Cosine Functions

Periodic Functions

Before we look at graphs of the sine and cosine functions, let's review some important terms. The trigonometric functions are **periodic**, which means that they are functions "for which a specific horizontal shift, P, results in a function equal to the original function" [1]. In other words, periodic functions repeat over and over. Figure 1 shows an example of a periodic function that is **not** one of the trigonometric functions.

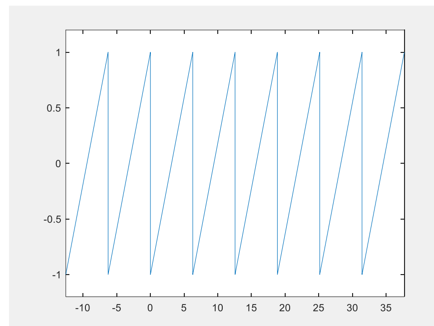

Figure 1. A Periodic Function

The horizontal shift P is called the **period**, and it is "the shortest interval over which a function completes one full cycle" [1]. In equation form, a periodic function $f(x)$ adheres to the following rule for all x values in its domain:

$$f(x+P)=f(x).$$

The Parent Sine and Cosine Functions

In contrast to the sawtooth function in Figure 1, the sine and cosine functions oscillate smoothly like waves. See Figure 2 and Figure 3 for graphs of the sine and cosine functions.

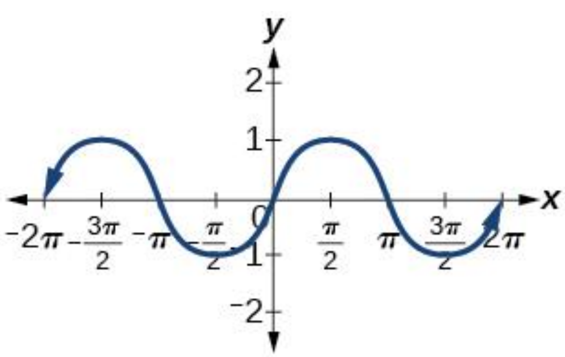

Figure 2. Graph of $y = \sin x$
Source: [1]

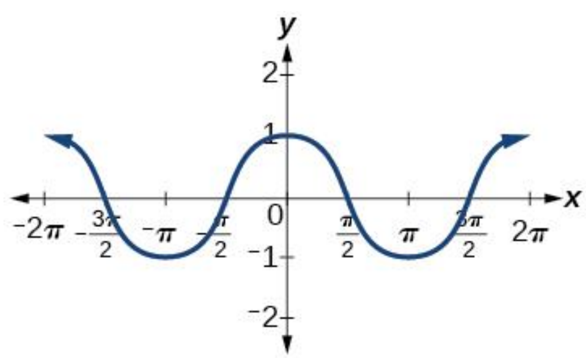

Figure 3. Graph of $y = \cos x$
Source: [1]

Note that they both have a period of 2π. See Figure 4 for other characteristics of sine and cosine.

CHARACTERISTICS OF SINE AND COSINE FUNCTIONS

The sine and cosine functions have several distinct characteristics:

- They are periodic functions with a period of 2π.
- The domain of each function is $(-\infty, \infty)$ and the range is $[-1, 1]$.
- The graph of $y = \sin\ x$ is symmetric about the origin, because it is an odd function.
- The graph of $y = \cos\ x$ is symmetric about the y-axis, because it is an even function.

Figure 4. Characteristics of Sine and Cosine Functions
Source: [1]

Sinusoidal Functions

A function that can be described as a combination of transformations of the sine or cosine function is called a **sinusoidal function** or simply a sinusoid. Sinusoids have the general form

$$y = A\sin(Bx - C) + D \text{ or } y = A\cos(Bx - C) + D.$$

Because cosine and sine are shifted versions of each other, the equation for a sinusoid can be written with cosine or sine. You can check if two equations for a sinusoid are equivalent using the cofunction identities from section 5.4.

In addition to the period, which is $2\pi/|B|$, sinusoids have a few other notable characteristics:

- **midline** – the horizontal line through the middle of the graph, $y = D$
- **amplitude** – "the vertical height from the midline", $|A|$ [1]
- **phase shift** – "the horizontal displacement of the basic sine or cosine function", C/B [1]

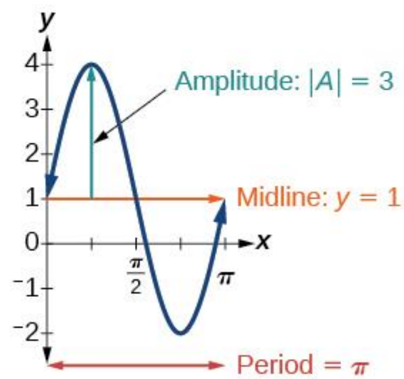

Figure 5. Characteristics of Sinusoids
Source: [1]

An example sinusoid with its midline, amplitude, and period labeled is shown in Figure 5. The phase shift is not shown because it changes based on whether we use a sine or cosine function to describe the wave. If we use a sine function, the phase shift is zero, but if we use a cosine function, the phase shift is $\pi/4$.

To graph a sinusoid, follow these steps:

1. If necessary, write the function in the form $y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$.
2. Determine its amplitude, $|A|$.
3. Determine its period, $P = \frac{2\pi}{|B|}$.
4. Graph the function $y = A\sin(Bx)$ or $y = A\cos(Bx)$.
5. Shift the graph of $y = A\sin(Bx)$ or $y = A\cos(Bx)$ left or right according to the phase shift, $\frac{C}{B}$.
6. Shift the graph from step 5 up or down according to the vertical shift, D.
7. If A is negative, reflect the graph across the x-axis. (B is not usually negative, but a negative B value would cause a reflection across the y-axis.)

To graph $y = A\sin(Bx)$ for step 4 above, follow these steps:

1. Plot a point at the origin.
2. Plot points on the x-axis at $x = \pm\frac{P}{2}, \pm P, \pm\frac{3P}{2}, \pm 2P, \dots$
3. Plot points with the y-value $\pm A$ at $x = \pm\frac{P}{4}, \pm\frac{3P}{4}, \pm\frac{5P}{4}, \pm\frac{7P}{4}, \dots$ Start with A and alternate back and forth with $-A$. (If A is positive, you will get a peak first, and if A is negative, you will get a trough first.)
4. Draw a sinusoidal curve through the points.

To graph $y = A\cos(Bx)$ for step 4 above, follow these steps:

1. Plot a point $(0, A)$. (If A is positive, this will be a peak, and if A is negative, this will be a trough.)
2. Plot points with the y-value $\pm A$ at $x = \pm\frac{P}{2}, \pm P, \pm\frac{3P}{2}, \pm 2P, \dots$ Start with $-A$ and alternate back and forth with A.
3. Plot points on the x-axis at $x = \pm\frac{P}{4}, \pm\frac{3P}{4}, \pm\frac{5P}{4}, \pm\frac{7P}{4}, \dots$
4. Draw a sinusoidal curve through the points.

6.2 Graphs of the Other Trigonometric Functions

Just like sine and cosine, the other trigonometric functions repeat. The graphs of tangent and cotangent are very similar because they are reciprocal functions. See Figure 6 and Figure 7.

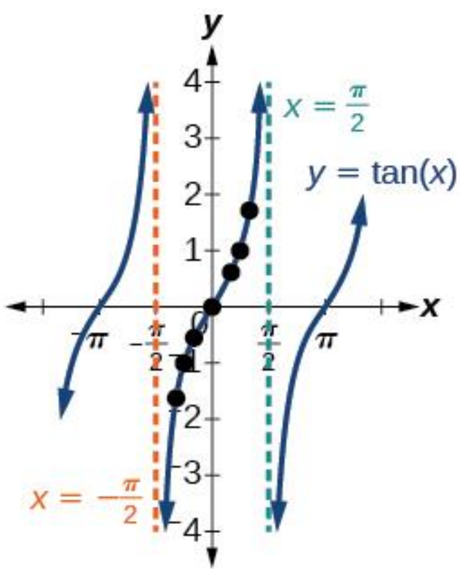

Figure 6. Graph of $y = \tan x$
Source: [1]

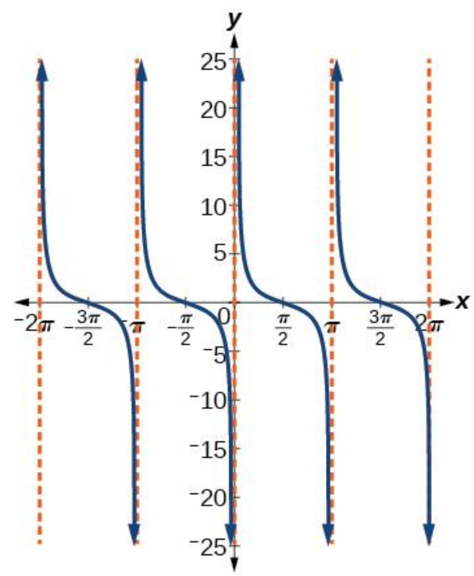

Figure 7. Graph of $y = \cot x$
Source: [1]

Note that tangent and cotangent's periods are π, and cotangent has asymptotes where tangent has zeros and vice versa [1]. Also, the parent tangent function crosses through $y = \pm 1$ at $x = \pm\pi/4$.

The graphs of secant and cosecant are also very similar because they are reciprocal functions of cosine and sine, respectively, and cosine and sine are very similar. See Figure 8 and Figure 9.

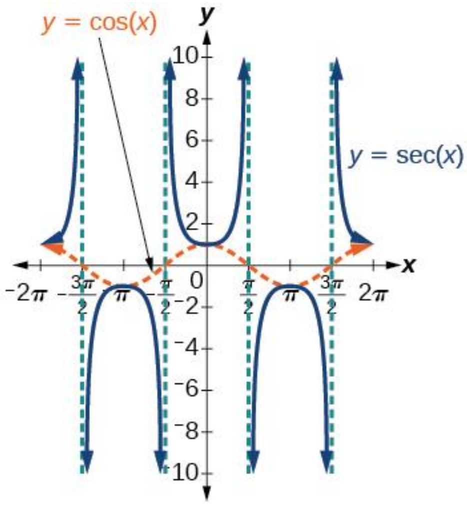

Figure 8. Graph of $y = \sec x$
Source: [1]

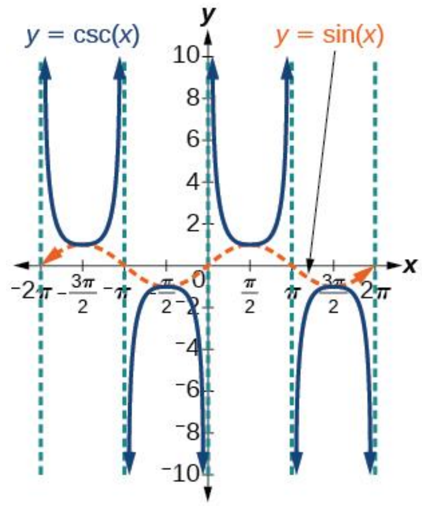

Figure 9. Graph of $y = \csc x$
Source: [1]

Secant's asymptotes occur where cosine is zero and cosecant's asymptotes occur where sine is zero.

Transformations of tangent, cotangent, secant, and cosecant can be graphed in a similar manner to transformations of sine and cosine.

Reminders

I hope that the Precalculus resources I shared with you this semester were helpful and efficient in addressing your academic needs. As we approach the end of the fall semester, I would like to let you know that there will be one more resource for this class next week, which will help you review the material for your final exam. All resources for this semester can be found here: https://www.baylor.edu/support_programs/index.php?id=967950. Also, please do not forget that the week of November 16-20 is the last week where group tutoring sessions will take place. If you wish to attend the last session, please make sure you check out the tutoring center website (https://www.baylor.edu/tutoring) and follow the instructions to register for group tutoring.

Here are a few reminders about Precalculus opportunities:

- The last group tutoring session for Precalculus is on Tuesday, November 17th from 5:00 to 6:00 p.m. through Microsoft Teams.
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/support_programs/index.php?id=955694.
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at https://www.baylor.edu/tutoring.

References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: https://openstax.org/details/books/precalculus