

# MTH 1320 – PRECALCULUS

## FALL 2020 WEEK 12 RESOURCE

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Welcome back, Precalculus scholars! This resource focuses on material covered in the twelfth week of classes, namely trigonometric functions and right triangle trigonometry from sections 5.3-5.4 of OpenStax's *Precalculus*. Please refer to previous resources if you would like to review other topics.

**Don't forget to sign up for Group Tutoring on Tuesdays at 5:00!**

### 5.3 The Other Trigonometric Functions

In section 5.2, you learned about the two most common trigonometric functions: sine and cosine. There are four other trigonometric functions: tangent, secant, cosecant, and cotangent. Like sine and cosine, these functions are defined with respect to a point  $(x, y)$  on the unit circle (circle with radius 1 centered on the origin) [1]. Figure 1 shows an angle  $t$  with the tip of its terminal side touching the unit circle at a point  $(x, y)$ . With this figure in mind, look at Table 1 for a summary of all six trigonometric functions. (Note that I used  $\theta$  for the angle instead of  $t$ .)

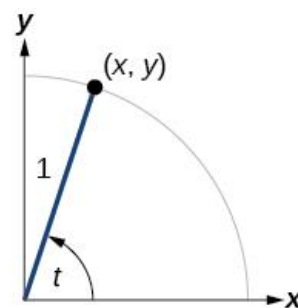


Figure 1. A Point on the Unit Circle  
Source: [1]

**Table 1. Summary of the Trigonometric Functions**

Name	Notation	Definition in Terms of Coordinates on the Unit Circle	Relationship to Other Trigonometric Functions
Cosine	$\cos \theta$	$x$	
Sine	$\sin \theta$	$y$	
Tangent	$\tan \theta$	$\frac{y}{x}, x \neq 0$	$\frac{\sin \theta}{\cos \theta}$
Secant	$\sec \theta$	$\frac{1}{x}, x \neq 0$	$\frac{1}{\cos \theta}$
Cosecant	$\csc \theta$	$\frac{1}{y}, y \neq 0$	$\frac{1}{\sin \theta}$
Cotangent	$\cot \theta$	$\frac{x}{y}, y \neq 0$	$\frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$

The relationships in the rightmost column of Table 1 are called the **fundamental identities**. **Either a point on the unit circle or the values of sine and cosine for a particular angle can be used to find the values of the other four trigonometric functions.** Additionally, the fundamental identities can be used to simplify an expression involving trigonometric functions.

## Using Reference Angles to Evaluate Trigonometric Functions

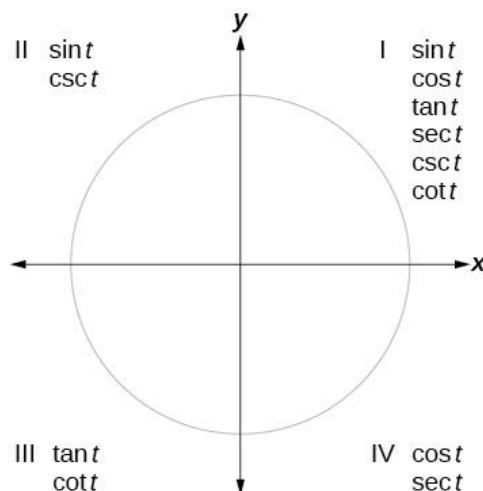
Like I said last week, it would be helpful for you to memorize the sine and cosine values of commonly used angles (i.e.  $\pi/6, \pi/4, \pi/3$ ) in the first quadrant of the coordinate plane. Once you know these values, you can determine the outputs of the trigonometric functions at common angles in the other quadrants using reference angles. See Figure 2 for instructions on how to use reference angles to evaluate the trigonometric functions.

**Given an angle not in the first quadrant, use reference angles to find all six trigonometric functions.**

1. Measure the angle formed by the terminal side of the given angle and the horizontal axis. This is the reference angle.
2. Evaluate the function at the reference angle.
3. Observe the quadrant where the terminal side of the original angle is located. Based on the quadrant, determine whether the output is positive or negative.

**Figure 2. Using Reference Angles to Evaluate Trigonometric Functions**  
Source: [1]

To “determine whether the output is positive or negative”, as mentioned in step 3 of Figure 2, we need to know the signs of the trigonometric functions in each quadrant of the coordinate plane. Figure 3 depicts which trigonometric functions are positive in each quadrant. (The functions that are not listed in each quadrant are negative.) **Note that all the trigonometric functions are positive in quadrant I.**



**Figure 3. Trigonometric Functions that are Positive in Each Quadrant**

## Evaluating Even and Odd Trigonometric Functions

Recall from section 1.5 that **even functions** are symmetric about the  $y$ -axis, and for them it holds true that  $f(-x) = f(x)$ . In contrast, **odd functions** are symmetric about the origin, and for them it holds true that  $f(-x) = -f(x)$ .

All trigonometric functions are symmetric in some way. In other words, if we evaluate a trigonometric function at a positive angle and a negative angle of the same magnitude, the absolute values of the two outputs will be the same. Whether each trigonometric function is even or odd can be determined by

examining the unit circle. The result of this examination is that cosine and secant are even, while the rest of the trigonometric functions are odd. Therefore, we have the following relationships:

- $\cos(-\theta) = \cos \theta$
- $\sin(-\theta) = -\sin \theta$
- $\tan(-\theta) = -\tan \theta$
- $\sec(-\theta) = \sec \theta$
- $\csc(-\theta) = -\csc \theta$
- $\cot(-\theta) = -\cot \theta$

#### Alternate Forms of the Pythagorean Identity

In section 5.2, we learned about the Pythagorean Identity:  $\cos^2 \theta + \sin^2 \theta = 1$ . Now that we know about all six trigonometric functions, there are two other forms of the Pythagorean Identity that may be useful to you:

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

The alternate forms can be derived by dividing the original form by  $\cos^2 \theta$  and  $\sin^2 \theta$ , respectively. Try it for yourself!

Given one trigonometric function value, you can use the Pythagorean Identity to help find the other trigonometric function values.

### 5.4 Right Triangle Trigonometry

Some important applications of trigonometry involve right triangles. So far, we have defined the trigonometric functions in terms of a point on the unit circle, but we can also define them based on the sides of a right triangle [1]. This makes the trigonometric functions much more versatile. The sides of a right triangle are called the hypotenuse, adjacent side, and opposite side. The hypotenuse is always the angle opposite to the right angle. The opposite and adjacent sides, however, vary based on which acute angle in the triangle we want to examine. The opposite side is the side opposite of the acute angle in question. The adjacent side is the side adjacent to (next to) the acute angle in question. See Figure 4.

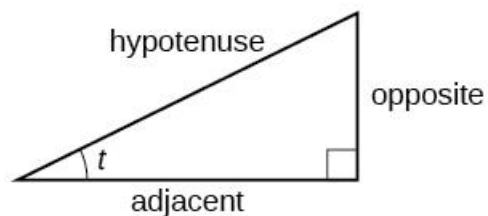


Figure 4. The Sides of a Right Triangle  
Source: [1]

Knowing these terms for the sides of a right triangle, we can now learn how trigonometric functions of an acute angle  $\theta$  are related to these sides.

- $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

A way to remember these relationships is the mnemonic Soh Cah Toa:

- **Soh:** Sine is opposite over hypotenuse.
- **Cah:** Cosine is adjacent over hypotenuse.
- **Toa:** Tangent is opposite over adjacent.

Note that the reciprocal relationship between secant, cosecant, and cotangent and cosine, sine, and tangent, respectively, still holds. Therefore, definitions for the other trigonometric functions in terms of the sides of a right triangle could be derived using the above definitions for sine, cosine, and tangent.

### Cofunction Identities

Some useful identities called the **cofunction identities** can be derived because the two acute angles in a right triangle are complementary.

- $\cos \theta = \sin \left( \frac{\pi}{2} - \theta \right)$
- $\sin \theta = \cos \left( \frac{\pi}{2} - \theta \right)$
- $\tan \theta = \cot \left( \frac{\pi}{2} - \theta \right)$
- $\cot \theta = \tan \left( \frac{\pi}{2} - \theta \right)$
- $\sec \theta = \csc \left( \frac{\pi}{2} - \theta \right)$
- $\csc \theta = \sec \left( \frac{\pi}{2} - \theta \right)$

### Reminders

Thank you for reading this resource! Here are a few reminders about other Precalculus opportunities:

- Group Tutoring for Precalculus is on Tuesdays from 5:00 to 6:00 p.m. through Microsoft Teams. If you have not received an email inviting you to sign up, and you want to attend, please email me at [Sydney\\_Schirner1@baylor.edu](mailto:Sydney_Schirner1@baylor.edu), and I will make sure you are included!
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at [https://www.baylor.edu/support\\_programs/index.php?id=955694](https://www.baylor.edu/support_programs/index.php?id=955694).
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at <https://www.baylor.edu/tutoring>.

### References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: <https://openstax.org/details/books/precalculus>