

# MTH 1320 – PRECALCULUS

## FALL 2020 WEEK 9 RESOURCE

By Sydney Schirner

Welcome back, Precalculus scholars! This resource focuses on material covered in the ninth week of classes, namely logarithmic functions from Sections 4.3-4.4 of OpenStax's *Precalculus*. Please refer to previous resources if you would like to review other topics. **Don't forget to sign up for Group Tutoring on Tuesdays at 5:00!**

### 4.3 Logarithmic Functions

Last week, we discussed exponential functions, and you may be wondering, "What is the inverse of an exponential function?" The inverse of the exponential function  $f(x) = b^x$  is  $f^{-1}(x) = \log_b(x)$ . This function is called a **logarithm** or a logarithmic function, and it has the following definition:

$$y = \log_b(x) \text{ is equivalent to } b^y = x$$

under the following conditions:

- The input  $x$  must be greater than zero.
- The base  $b$  must be greater than zero (as with exponential functions).
- The base  $b$  must NOT be equal to 1 (as with exponential functions).

**In words, the logarithm (or log)  $y$  with base  $b$  of  $x$  is "the exponent to which  $b$  must be raised to get  $x$ " [1].** Also, because the logarithm is the inverse of the exponential, its domain is the range of the exponential, and its range is the exponential's domain.

- Domain:  $(0, \infty)$
- Range:  $(-\infty, \infty)$

Because logarithms are equivalent to exponentials, we can convert logarithms to exponentials and vice versa. Rewriting a logarithm in exponential form makes it easier to evaluate a logarithm because we can ask ourselves, "To what exponent should  $b$  be raised in order to get  $x$ ?" [1].

There are two logarithms that are frequently used and have special names:

- **A common logarithm** has a base of 10 and is often written  $\log(x)$ . **If you see a logarithm without a base, it is implied that it is a common or base-10 logarithm.**
- **A natural logarithm** has a base of  $e$  and is written  $\ln(x)$ .

### 4.4 Graphs of Logarithmic Functions

#### Finding a Logarithm's Domain

If a logarithm's argument (the expression inside the parentheses) is not simply  $x$ , the domain will be restricted based on that argument. Because the logarithm is undefined for input values less than or equal to zero, we can find the domain of a logarithmic function through the following steps:

1. **Set the expression inside the parentheses (argument) greater than zero.**
2. **Solve the resulting inequality for  $x$ .**

## Graphing a Logarithmic Function

The parent logarithmic function,  $f(x) = \log_b(x)$ , is graphed as shown in Figure 1. Note these features:

- a vertical asymptote at  $x = 0$
- an  $x$ -intercept at  $(1, 0)$
- a “key point” at  $(b, 1)$

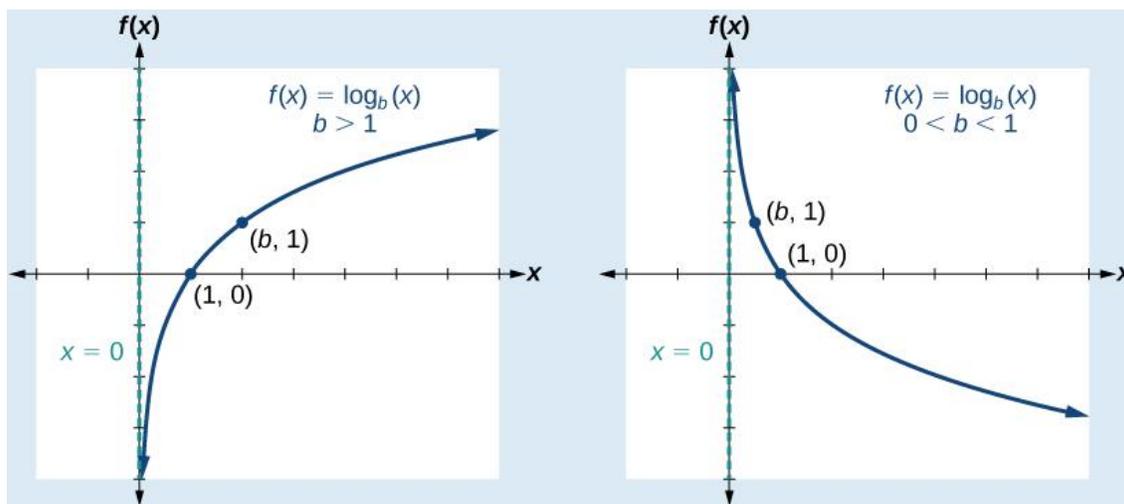


Figure 1. Graphs of the Parent Logarithmic Function  
Source: [1]

We can see that if the base is greater than 1, the graph will be increasing, while if the base is between zero and one, the graph will be decreasing. We can verify this by looking at the equivalent exponential  $b^y = x$ . If we have a base of  $b = 2 > 1$ , we must increase the  $y$  values to get greater  $x$  values. However, if we have a base of  $b = \frac{1}{2} < 1$ , we must use more negative  $y$  values to get greater  $x$  values. For example,

$$\left(\frac{1}{2}\right)^{-2} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$$

We must have a negative exponent ( $y$  value) to change the fraction into a number greater than 1.

To draw a graph of the parent logarithm, we can follow the process shown in Figure 2.

### HOW TO

Given a logarithmic function with the form  $f(x) = \log_b(x)$ , graph the function.

1. Draw and label the vertical asymptote,  $x = 0$ .
2. Plot the  $x$ -intercept,  $(1, 0)$ .
3. Plot the key point  $(b, 1)$ .
4. Draw a smooth curve through the points.
5. State the domain,  $(0, \infty)$ , the range,  $(-\infty, \infty)$ , and the vertical asymptote,  $x = 0$ .

Figure 2. Steps to Graph the Parent Logarithmic Function  
Source: [1]

All other logarithmic functions can be graphed as transformations of the parent logarithmic function. **Note that a logarithm that is horizontally shifted, like  $f(x) = \log(x - c)$ , will have the vertical asymptote  $x = c$ .** A horizontal stretch/compression can also affect the vertical asymptote.

### Example Problems

1. Rewrite the equation  $y = \ln(x)$  in exponential form.  
Recall that  $\ln(x)$  is a natural log, which has the base  $e$ , so  $b = e$ . Therefore, the exponential form of the equation is

$$e^y = x.$$

2. Solve the equation  $\log_3 x = -2$  for  $x$  by rewriting it in exponential form.  
Note that  $b = 3$  and  $y = -2$ . Therefore,

$$x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$$

3. Find the domain of the logarithmic function  $f(x) = \log_2(-3x + 9) + 5$ .
  - a. Set the expression inside the parentheses (argument) greater than zero.

$$-3x + 9 > 0$$

- b. Solve the resulting inequality for  $x$ .  
Subtract 9 from both sides.

$$-3x > -9$$

Divide by  $-3$  and flip the inequality.

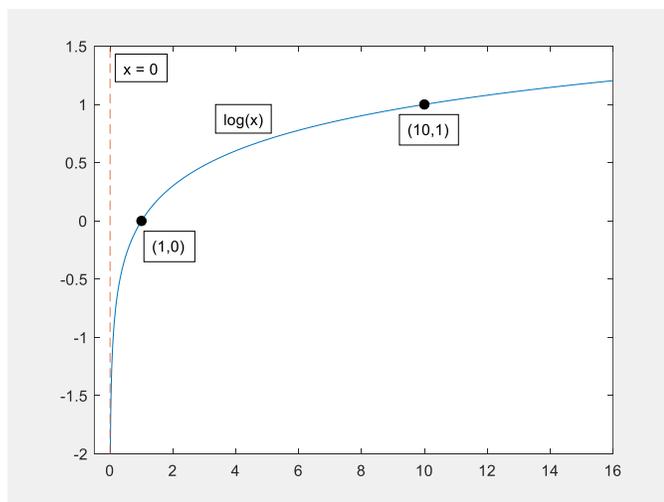
$$x < \frac{-9}{-3}$$

$$x < 3$$

The domain is  $(-\infty, 3)$ .

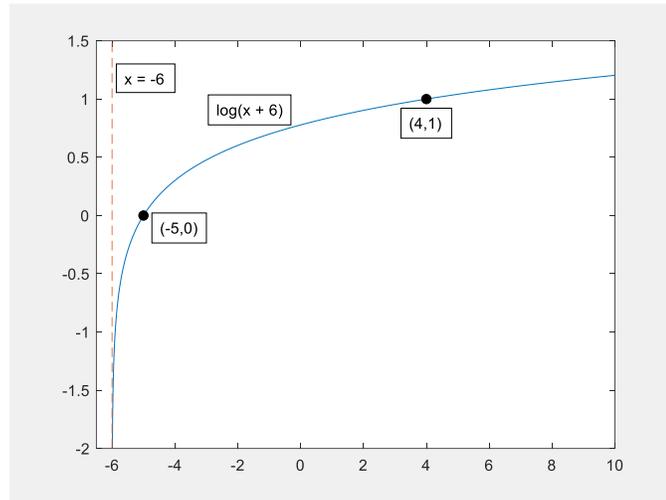
4. Sketch the graph of the function  $g(x) = \log(6 - 3x) + 1$  [1].

Let's start by **graphing the parent function**,  $\log(x)$ . The  $x$ -intercept will be  $(1, 0)$ , as for any parent log function, and the key point will be  $(b, 1) = (10, 1)$ .

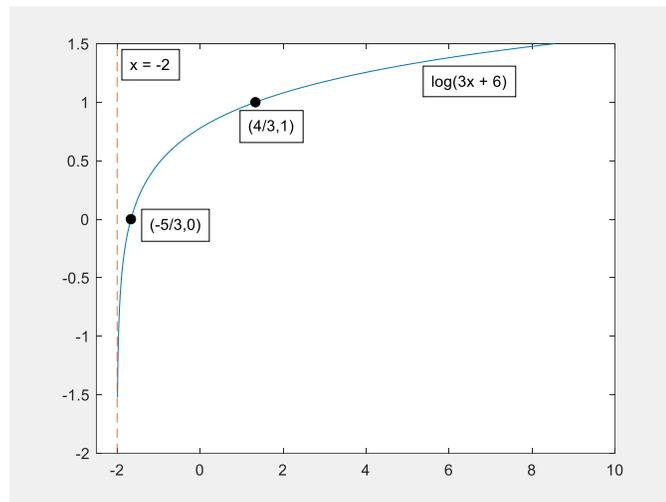


Now let's **perform the horizontal shift** to get  $\log(x + 6)$ . Because the parent function  $\log(x)$  is shifted 6 units to the left, the vertical asymptote will also be shifted 6 units to the left. Therefore, the vertical asymptote will be at  $x = -6$ . The  $x$ -intercept will also be shifted

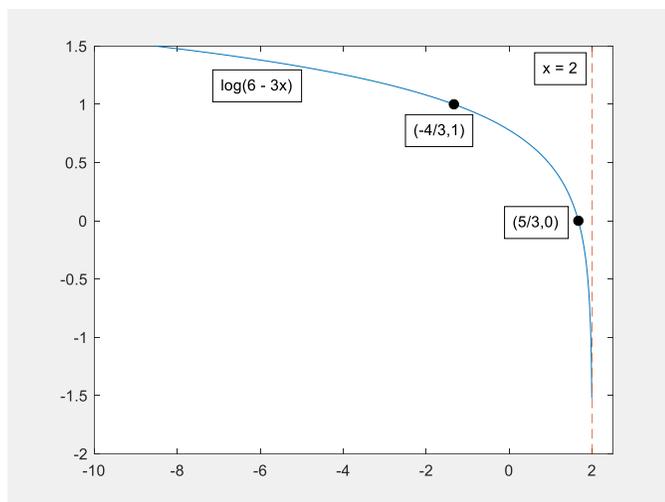
horizontally to  $(1 - 6, 0) = (-5, 0)$ , and similarly the key point is now  $(10 - 6, 1) = (4, 1)$ . Below is the graph of  $\log(x + 6)$ .



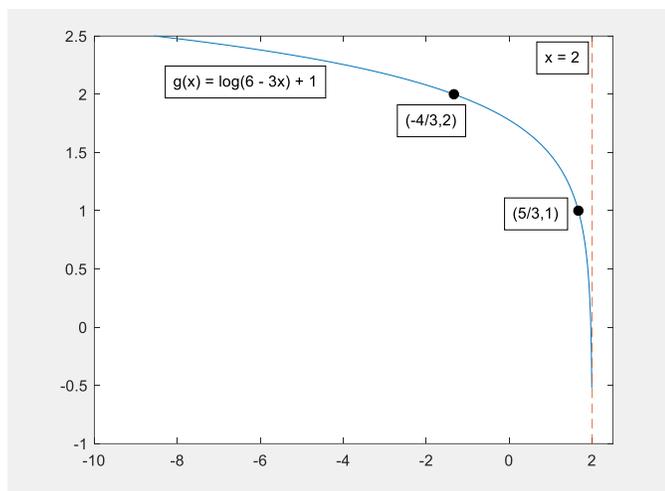
Next, let's **compress the graph horizontally** by 3 units to get  $\log(3x + 6)$ . To implement the compression, we must divide each  $x$ -coordinate by 3. This will change the vertical asymptote to  $x = -\frac{6}{3} = -2$ , the  $x$ -intercept to  $(-\frac{5}{3}, 0)$ , and the key point to  $(\frac{4}{3}, 1)$ .



To complete the necessary horizontal transformations, we must **reflect the graph across the  $y$ -axis** to get  $\log(-3x + 6) = \log(6 - 3x)$ . In other words, we must negate each  $x$ -coordinate. This will change the vertical asymptote to  $x = -(-2) = 2$ , the  $x$ -intercept to  $(-\left(-\frac{5}{3}\right), 0) = (\frac{5}{3}, 0)$ , and the key point to  $(-\frac{4}{3}, 1)$ .



Lastly, we must **shift the graph up** by 1 unit to get  $g(x) = \log(6 - 3x) + 1$ . Each  $y$ -coordinate should be increased by 1. Therefore, we have the points  $(\frac{5}{3}, 0 + 1) = (\frac{5}{3}, 1)$  and  $(-\frac{4}{3}, 1 + 1) = (-\frac{4}{3}, 2)$ . The vertical asymptote is unaffected. The final graph is shown below.



## Reminders

Thank you for reading this resource! Here are a few reminders about other Precalculus opportunities:

- Group Tutoring for Precalculus is on Tuesdays from 5:00 to 6:00 p.m. through Microsoft Teams. If you have not received an email inviting you to sign up, and you want to attend, please email me at [Sydney\\_Schirner1@baylor.edu](mailto:Sydney_Schirner1@baylor.edu), and I will make sure you are included!
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at [https://www.baylor.edu/support\\_programs/index.php?id=955694](https://www.baylor.edu/support_programs/index.php?id=955694).
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at <https://www.baylor.edu/tutoring>.

## References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: <https://openstax.org/details/books/precalculus>