

MTH 1320 – PRECALCULUS

FALL 2020 WEEK 5 RESOURCE

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Welcome back, Precalculus scholars! This resource focuses on material covered in the fifth week of classes, namely topics from sections 3.2-3.3 of OpenStax's *Precalculus*. If your class is covering linear functions (Chapter 2) please refer to the Week 4 Resource. Don't forget to sign up for Group Tutoring on Tuesdays at 5:00!

3.2 Quadratic Functions

A quadratic function is “a function of degree two” [1]. Its graph is a parabola. For example, the toolkit function $f(x) = x^2$ is the basic quadratic function. The **general form** of a quadratic function is

$$f(x) = ax^2 + bx + c,$$

where a , b , and c are real constants and $a \neq 0$. (If a were zero, the function would not be of degree 2.)

It is often easier to visualize a parabola when its equation is in **standard form**:

$$f(x) = a(x - h)^2 + k,$$

where (h, k) is the **vertex**, or lowest/highest point, of the parabola (see Figure 1). If $a > 0$, the parabola opens upward (like a smile), and if $a < 0$, the parabola opens downward (like a frown).

To determine the vertex of a parabola from the general form of the quadratic function,

1. Calculate the x -coordinate of the vertex using the formula

$$h = -\frac{b}{2a}$$

2. Evaluate $f(x)$ at $x = h$ to find k .

We often want to find the roots, or x -intercepts, of a quadratic function. There are a few methods to determine the roots, each of which is useful for various problems:

- When the function is easily factorable, factor it, set the factors equal to zero, and solve for x .
- When the quadratic function is in general form, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- Rewrite the quadratic function in standard form, set it equal to zero, and solve for x . (If you know the method of completing the square, this is very similar.)

3.3 Power Functions and Polynomial Functions

Power Functions

A power function is “a function with a single term that is the product of a real number...and a variable raised to a fixed real number” [1]. The number that multiplies the variable is called a **coefficient**. In equation form, power functions look like this:

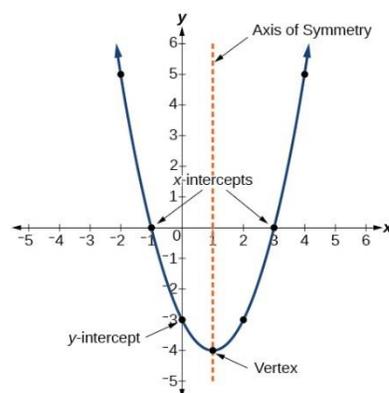


Figure 1. The Features of a Parabola
Source: [1]

$$f(x) = kx^p,$$

where k and p are both real numbers. The exponent p can be negative and does not have to be an integer; it can be any real number.

We often want to know what the graph of a function generally looks like, and part of this is the function's end behavior. The **end behavior** of a function is what happens to its outputs as its inputs decrease towards negative infinity and increase towards positive infinity. We often use the notation, "as $x \rightarrow \infty$ (or $-\infty$), $f(x) \rightarrow _$ " when talking about end behavior. (This is read, "as x approaches infinity (or negative infinity), $f(x)$ approaches _____.")

For a power function with a non-negative integer exponent, the end behavior possibilities are limited to $\pm\infty$, and we can determine which based on the sign of the coefficient k and whether the power is even or odd. See Figure 2 for a summary of the various end behaviors.

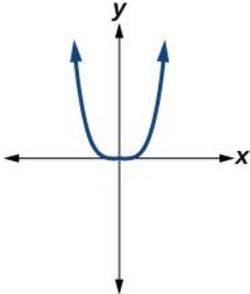
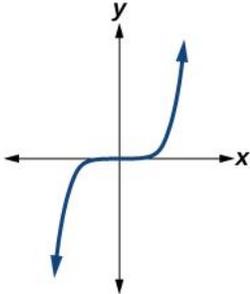
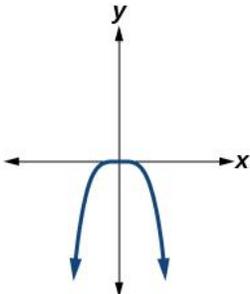
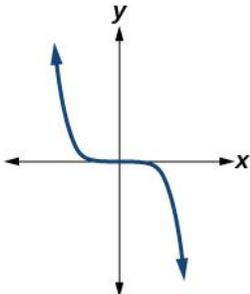
	Even power	Odd power
Positive constant $k > 0$	 <p>$x \rightarrow -\infty, f(x) \rightarrow \infty$ and $x \rightarrow \infty, f(x) \rightarrow \infty$</p>	 <p>$x \rightarrow -\infty, f(x) \rightarrow -\infty$ and $x \rightarrow \infty, f(x) \rightarrow \infty$</p>
Negative constant $k < 0$	 <p>$x \rightarrow -\infty, f(x) \rightarrow -\infty$ and $x \rightarrow \infty, f(x) \rightarrow -\infty$</p>	 <p>$x \rightarrow -\infty, f(x) \rightarrow \infty$ and $x \rightarrow \infty, f(x) \rightarrow -\infty$</p>

Figure 2. End Behavior of Power Functions with Non-negative Integer Powers
Source: [1]

Additionally, power functions with even powers are symmetric about the y -axis (i.e. they are **even functions**), and those with odd powers are symmetric about the origin (i.e. they are **odd functions**).

Polynomial Functions

The general form of a polynomial function is

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are real coefficients, and $n, n - 1, \dots, 2, 1, 0$ are non-negative integer powers. a_n cannot be zero because that would change the degree of the polynomial, but any other coefficient can be zero. Also, note that the polynomial's **terms**, $a_i x^i$ (i.e. the things separated by the plus signs), must be organized with the powers going from greatest to least for it to be in general form.

Note that there is overlap between power functions and polynomial functions; polynomial functions that have $a_{n-1} = a_{n-2} = \cdots = a_2 = a_1 = a_0 = 0$ (i.e. all the coefficients except the first one are zero) are also power functions.

A polynomial's **degree** is "the highest power of the variable that occurs in the polynomial" or n in the equation above [1]. The first term ($a_n x^n$) when the equation is in general form is also known as the **leading term**, and the leading term's coefficient (a_n) is called the **leading coefficient**. See Figure 3.

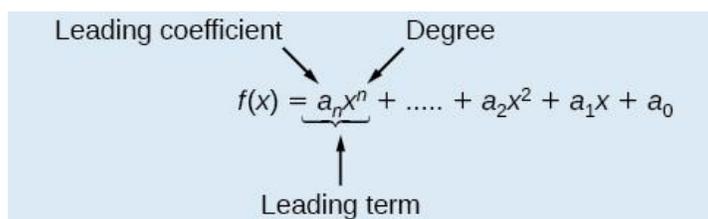


Figure 3. Terminology for Polynomials
Source: [1]

The end behavior of a polynomial function is determined by the end behavior of its leading term. In other words, to determine a polynomial's end behavior:

1. Find the term with the highest power and look at it as a power function.
2. Determine that power function's end behavior using Figure 2.
3. Recognize that this end behavior is the same as the overall polynomial's end behavior.

You might be wondering, "What does the rest of the polynomial's graph look like?" Although it is difficult to completely know what the graph looks like without calculus techniques, we can determine the number of x -intercepts (or roots or zeros) and **turning points** (or extrema) of the graph. If a polynomial has a degree of n , it will have no more than n x -intercepts and no more than $n - 1$ turning points. For example, quadratic functions, from section 3.2, can have up to 2 roots and 1 turning point because they are polynomials of degree 2.

We can determine where the x -intercepts of polynomials are by setting the function equal to zero and solving for x , as you have done for quadratic functions. We can also determine the single y -intercept of a polynomial by evaluating the function at zero.

Example Problems

1. Write the following quadratic function in standard form: $f(x) = 2x^2 + 4x + 1$. Use the standard form to find the function's x -intercepts.

Find h and k .

$$h = -\frac{b}{2a} = -\frac{4}{2 * 2} = -\frac{4}{4} = -1$$
$$k = f(-1) = 2(-1)^2 + 4(-1) + 1 = 2(1) - 4 + 1 = -1$$

Write the quadratic function in standard form.

$$f(x) = a(x - h)^2 + k$$
$$f(x) = 2(x - (-1))^2 - 1$$
$$f(x) = 2(x + 1)^2 - 1$$

Set the function equal to zero and solve for x to find the x -intercepts.

$$f(x) = 2(x + 1)^2 - 1 = 0$$
$$2(x + 1)^2 = 1$$
$$(x + 1)^2 = \frac{1}{2}$$
$$x + 1 = \pm \sqrt{\frac{1}{2}}$$
$$x = \pm \sqrt{\frac{1}{2}} - 1$$

Reminders

Thank you for reading this resource! Here are a few reminders about other Precalculus opportunities:

- Group Tutoring for Precalculus is on Tuesdays from 5:00 to 6:00 p.m. through Microsoft Teams. If you have not received an email inviting you to sign up, and you want to attend, please email me at Sydney_Schirner1@baylor.edu, and I will make sure you are included!
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at https://www.baylor.edu/support_programs/index.php?id=955694.
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at <https://www.baylor.edu/tutoring>.

References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: <https://openstax.org/details/books/prec calculus>