

# MTH 1320 – PRECALCULUS

## FALL 2020 WEEK 1 RESOURCE

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Welcome, Precalculus scholars! I'm Sydney Schirner, a senior Electrical & Computer Engineering major and the Master Tutor for Precalculus. This document is the first Fall 2020 Precalculus resource and focuses on material covered in the third week of classes. The resource will discuss topics from sections 1.4-2.1 of OpenStax's *Precalculus*. Some classes are having their first test this week, so sic 'em bears!

### 1.4 Composition of Functions

Two functions that have the same kinds of inputs and same kinds of outputs can be combined using algebraic operations (+, −, ×, ÷), just like you would combine “like terms” in an equation. There are special notations for algebraic combinations of functions. For two functions  $f(x)$  and  $g(x)$  with real number outputs, we can define the following new functions:

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

We can also combine two functions using composition of functions. **Composition of functions** is “the process of combining functions so that the output of one function becomes the input of another” [1]. The resulting function is called a **composite function**. The notation for this combination is

$$(f \circ g)(x) = f(g(x)).$$

When evaluating a composite function, we start by evaluating the inner function  $g(x)$  at an input  $x$ . Then, the inner function's output  $g(x)$  becomes the input to the outer function  $f(x)$ . Note that this order is important.  $f(g(x))$  is not necessarily equal to  $g(f(x))$ !

To find the domain of a composite function  $(f \circ g)(x)$ , we first determine the domains of  $g(x)$  and  $f(x)$  individually. Next, we must exclude any inputs  $x$  to the inner function that give an output  $g(x)$  outside of the domain of  $f(x)$ . In other words, the domain of  $(f \circ g)(x)$  contains the inputs  $x$  in the domain of  $g(x)$  for which  $g(x)$  is in the domain of  $f(x)$ .

### 1.5 Transformation of Functions

To model various real-world situations, we need many different functions. However, this does not mean we have to invent all those functions from scratch. We can instead **transform** functions we know, like the toolkit functions in Section 1.1. One function can be transformed into a new one using **transformations**. Transformations are either vertical (in the  $y$ -direction) or horizontal (in the  $x$ -direction). For a function  $f(x)$ , vertical transformations are applied to the output  $f(x)$ , while horizontal ones are applied to the input  $x$  (i.e. added inside the parentheses). In this section, I will use  $f(x)$  as the original function,  $c$  as a constant, and  $g(x)$  as the new function produced by a transformation.

A **shift** moves a function up, down, right, or left. You can shift a function by adding or subtracting a constant from the function's input or output. See Figures 1 and 2 for examples of shifts.

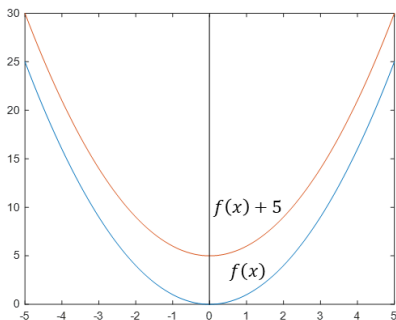


Figure 1. Vertical shift with  $c = 5$

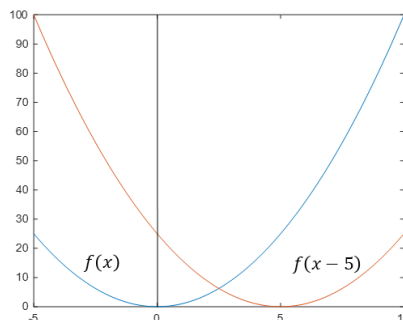


Figure 2. Horizontal shift with  $c = 5$

Table 1 summarizes the different kinds of shifts. Note that if there is a minus sign between  $x$  and the constant  $c$  inside the parentheses, the shift is to the right, NOT to the left!

Table 1. Shifts

Transformation	New Function	Meaning for $c > 0$	Meaning for $c < 0$
Vertical Shift	$g(x) = f(x) + c$	Shift up	Shift down
Horizontal Shift	$g(x) = f(x - c)$	Shift right	Shift left

A **reflection** flips the graph of a function over an axis. We can reflect a function by adding a negative sign in front of it (for a vertical reflection) or to its input (for a horizontal reflection). See Table 2.

Table 2. Reflections

Transformation	New Function	Meaning
Vertical Reflection	$g(x) = -f(x)$	Reflection about the $x$ -axis
Horizontal Reflection	$g(x) = f(-x)$	Reflection about the $y$ -axis

Lastly, we can apply a **stretch** or **compression** (multiplication by  $c$ ) by adding a coefficient. See Table 3.

Table 3. Stretches and Compressions

Transformation	New Function	Meaning for $c > 1$	Meaning for $0 < c < 1$
Vertical Stretch/Compression	$g(x) = cf(x)$	Stretch vertically	Compress vertically
Horizontal Stretch/Compression	$g(x) = f(cx)$	Compress horizontally	Stretch horizontally

## 1.7 Inverse Functions

For a function  $f(x)$ , its **inverse function** has inputs that are the outputs of  $f(x)$  and outputs that are the inputs of  $f(x)$ . The notation for the inverse is  $f^{-1}(x)$ . Note that the -1 is NOT an exponent.

A function and its inverse “undo each other.” This means that for a pair of inverse functions,

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x.$$

See Figure 3 for an example of a function and its inverse.

Again, the inputs of  $f(x)$  are the outputs of  $f^{-1}(x)$ , and the outputs of  $f(x)$  are the inputs of  $f^{-1}(x)$ . It then makes sense that the domain of  $f^{-1}(x)$  is the same as the range of  $f(x)$ , and the range of  $f^{-1}(x)$  is the same as the domain of  $f(x)$ .

Note that a function must be one-to-one (pass the horizontal line test) to have an inverse. Many functions that are not one-to-one can be restricted by narrowing their domains to a portion where the function is one-to-one. Then, we know that the function has an inverse on that restricted domain.

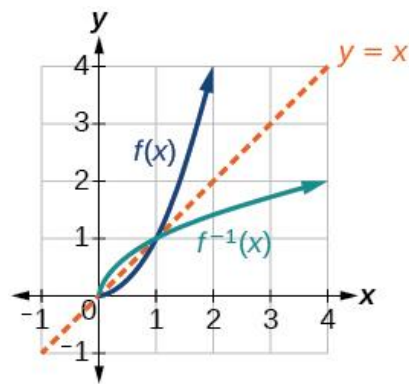


Figure 3.  $f(x)$  and  $f^{-1}(x)$

Source: [1]

To find the inverse of a one-to-one function,

1. Solve the function for  $x$ .
2. Switch the  $x$ 's and  $y$ 's (or  $f(x)$ 's).

## 2.1 Linear Functions

A **linear function** is “a function with a constant rate of change, that is, a polynomial of degree 1” [1]. For example,  $f(x) = 5x + 4$  is a linear function because the highest power of  $x$  it has is  $x^1$  or just  $x$ . The graph of a linear function is always a straight line and therefore can be written in **slope-intercept form**:

$$f(x) = mx + b$$

where  $m$  is the **slope** or **rate of change** of the line and  $b$  is its **y-intercept**.

The slope of a linear function determines whether it is increasing, decreasing, or constant. For **increasing** functions, the outputs increase with the inputs, and the graph has a positive slope (see Figure 4(a)). For **decreasing** functions, the outputs decrease as the inputs increase, and the graph has a negative slope (see Figure 4(b)). A **constant** function ( $f(x) = c$ ) outputs only one value, no matter the input. Its graph is therefore a horizontal line with a slope of zero (see Figure 4(c)).

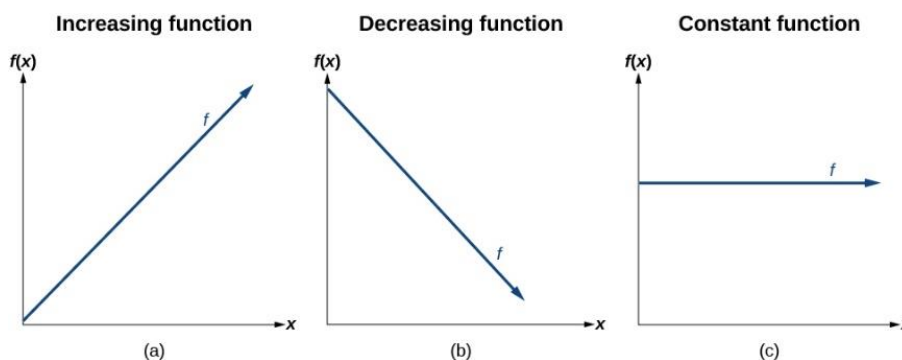


Figure 4. Different kinds of linear functions

Source: [1]

Given two points  $(x_1, y_1)$  and  $(x_2, y_2)$  from a linear function, its slope  $m$  can be calculated using

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1},$$

where  $\Delta$  stands for “change in.” The change in  $y$  is also known as the **rise**, and the change in  $x$  is also known as the **run**.

When given two points from a linear function, it is often convenient to write it in **point-slope form**:

$$y - y_1 = m(x - x_1)$$

where  $(x_1, y_1)$  is one of the given points and  $m$  is again the slope.

## Example Problems

1. If  $f(x) = 2x^2$  and  $g(x) = x - 3$ , find  $(f \circ g)(1)$ .

Method 1: Evaluate the inner function, and then evaluate the outer function.

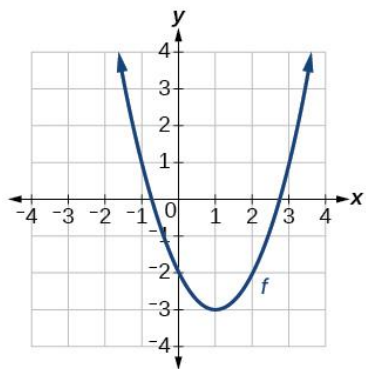
$$(f \circ g)(1) = f(g(1)) = f(1 - 3) = f(-2) = 2(-2)^2 = 8$$

Method 2: Find an equation for the composite function, and then evaluate it. (Method 2 is helpful when you need to evaluate at multiple values.)

$$(f \circ g)(x) = f(g(x)) = f(x - 3) = 2(x - 3)^2 = 2(x^2 - 6x + 9) = 2x^2 - 12x + 18$$

$$(f \circ g)(1) = 2(1)^2 - 12(1) + 18 = 2 - 12 + 18 = 8$$

2. Write an equation for the graphed function below.



Source: [1]

$f(x)$  is parabolic and therefore must be a transformation of  $g(x) = x^2$ . Since the vertex (lowest point) of  $g(x) = x^2$  is the origin,  $g(x)$  has been shifted down by 3 and to the right by 1. Therefore, the equation for the graphed function is

$$f(x) = g(x - 1) - 3 = (x - 1)^2 - 3.$$

3. Find the inverse of the function  $f(x) = x + 7$ .

Solve for  $x$ :  $y = x + 7 \rightarrow x = y - 7$

Switch  $x$  and  $y$ :  $y = x - 7$

Replace  $y$  with  $f^{-1}(x)$ :  $f^{-1}(x) = x - 7$

4. If the ordered pairs  $(2,1)$  and  $(5,7)$  are from the linear function  $f(x)$ , write an equation for  $f(x)$  in point-slope form.

Find the slope:  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 1}{5 - 2} = \frac{6}{3} = 2$

Choose one of the points: I'm going to choose  $(2,1)$ , but you could also choose  $(5,7)$ .

Write the equation:  $y - 1 = 2(x - 2)$

## Reminders

Thank you for reading this resource! Here are a few reminders about other Precalculus opportunities:

- Group Tutoring for Precalculus is on Tuesdays from 5:00 to 6:00 p.m. through Microsoft Teams. Precalculus students, you will receive an email notification a day or so before the session inviting you to request an appointment for it via Navigate (<https://baylor.campus.eab.com/>).
- The Tutoring Center has great Precalculus videos on YouTube! You can find links for them at [https://www.baylor.edu/support\\_programs/index.php?id=955694](https://www.baylor.edu/support_programs/index.php?id=955694).
- If you want individual help, you can also schedule a FREE 30-minute appointment with one of our awesome Precalculus tutors at <https://www.baylor.edu/tutoring>.

## References

[1] J. Abramson *et al.*, *Precalculus*. Houston, Texas, USA: Rice Univ., 2014. [Online]. Available: <https://openstax.org/details/books/prec calculus>