The particle nearest the center was perturbed with a gaussian force packet applied to it for several seconds in a number of different directions and contributed to the acceleration of the particles. The positions and velocities for all the particles were recorded for a few seconds after the kick is applied. These data are used to check localization by calculating the distance to the cyclic subspace.

Anderson Localization and the Spectral Method

- Spectral theory is a generalization of eigendecomposition based on the ability to decompose operators into outer products of "cobasis vectors".
- On a continuous basis, spectral theory allows for a change of measure in the completeness relation to include a weight function multiplying an eigenvalue function, on the discrete and absolutely continuous parts [2]:

\[
\int f d\mu = \int \left[ f(\delta(x - \lambda_1) + \delta(\xi - \lambda_2) + \cdots) + \nu(\xi) f(\xi) \right] d\xi
\]

- There is another "discrete-continuous" part of the measure that cannot be easily written down.
- For Schrodinger’s equation it is clear that the continuous part corresponds to extended states, and the discrete part to localized states.
- Vectors corresponding to the continuous part of the measure are “cyclic” in the hamiltonian. i.e a state |ψ⟩ is an extended state if: \(\text{span} \{\phi_i, H\phi_i, H^2\phi_i, \cdots\} = \mathbb{R}^{\infty}\) in [2]. This span is called the cyclic subspace.
- This makes sense - hamiltonian is the generator of time translation and in time evolving an extended state it will spread out in space like a gaussian wave packet. Here H has a continuous eigenspectra.
- Scaling theory solves the hamiltonian numerically with application of different boundary conditions (2016 Nobel Prize).
- Scaling theory appears to fail for 2D where it always predicts localization if there’s disorder. Mathematicians say it bounds the Hilbert space and may throw out some solutions.

Numerical experiment in Complex Plasma Crystals

- The fact scaling theory appears to fail in the 2D case is very relevant to 2D materials such as graphene.
- It is simple to create honeycomb lattice plasma crystal graphene analogues using PK-4 mechanism [4].
- PK-4 creates a plasma using RF frequency waves to ionize gas, and plastic spheres are introduced into the gas. They change, and a radial confinement E-field is applied by holding a parabolic indentation below the crystal at a certain potential, electrodes balance gravity.
- Crystal generation is modeled using the box_tree code developed by Derek Richardson and Lorin Matthews, with variation in confinements and particle sizes.
- A number of crystals at equilibrium with different disorders were generated.
- Disorder is measured by the "number of particles with a different number of nearest neighbours/the number of particles with 6 nearest neighbours" - called the defect in solid state.

![Gaussian Kick of .04 sec std .01 max .02 of crystalwith ω²=2500 (rad/Hz)](image)

![Plot of distance to the cyclic subspace for localized state with crystalline disorder](image)

References


And By NASA under Grant No. 1571701

Spectral Approach to Searching for Extended States in 2D Plasma Crystals
Forrest Guyton\textsuperscript{1,2}, Evdokiya Kostadinova\textsuperscript{2}, Lorin S Matthews\textsuperscript{2}, Truell Hyde\textsuperscript{2}
\textsuperscript{1}Physics, Applied Physics, and Astronomy, Rensselaer Polytechnic Institute, Jonsson Rowland Science Center, 110 Eighth Street Troy, New York 12180, USA
\textsuperscript{2}Center for Astrophysics, Space Physics, and Engineering Research
One Bear Place 97310, Waco, Texas, 76798-7310 USA
Classifying States with the Spectral Approach to Anderson Localization

F. Guyton, E. Kostadinova, L. Matthews, T. Hyde

Center for Astrophysics, Space Physics, & Engineering Research
Baylor University
Outline

Anderson Localization
Spectral Theory
Cyclic Subspace-Time Evolution of states
Numerical Methods
2d materials and plasma crystals
Data
Conclusion
Anderson Localization

Localization In tight binding model with random disorder
Extended states may become localized by self
interference from phase changed parts of the wave
reflected from crystal defects
Localization is defined as the wavefunction falling off in
space faster than exponential decay
Delocalization is falling off nearly linearly-Ohm's law
for Classical waves MIT transition occurs when $l \sim \lambda$
For particles: purely quantum effect- classical analog is
Einstein's brownian motion.

$$\vec{J} = -D \nabla \phi$$
The Anderson Hamiltonian

varying onsite energies and hopping potential has the same effect as varying of one of these variables

\[ \hat{H}_\epsilon = -\Delta + \sum_{i \in \Gamma} \epsilon_i \delta_i \mid \delta_i \rangle \]

\[ \hat{H} = \begin{pmatrix} \epsilon_1 & I & 0 & 0 \\ I & \epsilon_2 & I & 0 \\ 0 & I & \epsilon_3 & I \\ 0 & 0 & I & \epsilon_4 \end{pmatrix} \]
Scaling theory

Edwards/ Thouless-2016 Nobel prize

- Scaling theory-application of periodic/antiperiodic boundary conditions. Peaks that consistently show up are localized. Model scaled up.

- Appears to fail in 2D

- Application of boundary conditions bounds the Hilbert space - may throw out some solutions
Spectral Theory

Generalization of eigendecomposition to continuous basis
Can decompose operators into outer products of “cobasis vectors” (kets and bras)
Relies upon a change of measure in the completeness relation

\[ \int f d\mu = \int [f[\delta(\xi - \lambda_1) + \delta(\xi - \lambda_2) + \cdots] + w(\xi)f(\xi)] d\xi \]

- Measure decomposed into 3 parts
- Obvious application to Schrodinger’s equation
What part of the measure does an eigenvector correspond to?

It is shown generally that a vector corresponds to the continuous part if it is cyclic in the operator. In 2D,

\[
\text{span} \left\{ |\psi\rangle, \hat{H}|\psi\rangle, \hat{H}^2|\psi\rangle, \ldots \right\} = \mathbb{R}^2
\]

- makes sense - time translation
Numerical Method

At each time step, calculate the distance to the cyclic subspace at each timestep and determine if it goes to zero in the end.

\[ D_{\omega,c}^n = \sqrt{1 - \sum_{k=0}^{n} \frac{\langle m_k, v \rangle^2}{\| m_k \|_2^2}} \]

- Max distance normalized at 1
- \( m_k \) is the kth cyclic vector
- Grahm-Schmidt style orthogonal space of normalized vectors
2D materials

In the mid 2000’s we developed our first 2D materials (graphene). They have many applications. graphene is 2D graphite, it is an incredibly good barrier very strong very flexible

In experimentation, delocalization was observed in this material although scaling theory always predicts localization for all disorders
Plasma Crystals

It is very difficult to manufacture graphene, but plasma crystals can be created in a PK4 mechanism with the same honeycomb structure by putting a small parabolic indentation held at a certain potential below the dust particles to act as radial confinement with a linear restoring force.
Computer generated plasma crystals

A computer was used to generate plasma crystals at equilibrium with variation in confinements and particle size.

Then the particle nearest the center was kicked and the distance to the cyclic subspace at each timestep was calculated.
gaussian kick for .04 s at .016 m/s² directly up and to the left
An example of localization

Gaussian Kick of .04 sec std .01 max .02 of crystal with $\omega^2=2500 \text{ (rad/Hz)}^2$

Plot of distance to the cyclic subspace for localized state with crystalline disorder $W=0.0586$
calculated in the radius of 0.04 m

$\omega$ is the eigenfrequency of the confining E-field, which exerts a linear restoring force. $E=-kx$
An Example of Delocalized States

- Kick strength 0.015 m/s² in the 150deg direction
- Kick strength 0.015 m/s² in the 0deg direction
- Kick strength 0.016 m/s² in the 150deg direction
- Kick strength 0.016 m/s² in the 0deg direction
- Kick strength 0.015 m/s² in the down direction
- Kick strength 0.015 m/s² in the upleft direction
- Kick strength 0.016 m/s² in the down direction
- Kick strength 0.016 m/s² in the upleft direction
- Kick strength 0.015 m/s² in the neg30deg direction
- Kick strength 0.016 m/s² in the neg30deg direction
Conclusion

The spectral method is valuable in the study of Anderson localization in 2D by its capacity to find extended states without throwing out solutions by bounding the space. Plasma crystals are a useful 2D analogue of graphene. Extended states do exist in 2D materials with nonzero $W$. 
Thanks for your attention!
Questions