The Primes, Quantum Mechanics, Random Matrices and the Riemann Hypothesis

I will discuss one of the central problems in mathematics - the Riemann Hypothesis - which is related to the distribution of the prime numbers. In particular, I will describe highly speculative ideas which relate this problem to quantum mechanics. This has led to several new insights concerning long-standing statistical questions that arise in the theory of the primes, where mathematical physics plays a surprising role.