# Transverse Waves through one-dimensional Vertical Dust chains in a glass box

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Abstract—Vertical chains of micrometer-sized particles are created inside a small glass box inside an rf GEC reference cell. A powered probe attached to a Zyvex S100 Nanomanipulator perturbs these particles. Plasma parameters are studied by measuring the resonant frequency of a single particle by driving the probe potential as a square wave, the chain height as the DC bias changes, and the sheath edge height from the lower electrode as power increases. Dispersion relations are found for transverse chain waves created by a sinusoidal probe potential oscillation. A model is created to find the charge and Debye length needed for particles to levitate at their experimental positions.

*Index Terms*—Dust Chains, Dusty Plasmas, Glass Box, Transverse Wave.

## I. INTRODUCTION

DUSTY plasmas are plasmas with added micrometer-sized particles. Dusty plasmas are found in Saturn's rings and the semiconductor industry [1]. Using an rf powered GEC reference cell, dusty plasmas are created in the laboratory. The spacing between the powered lower and grounded upper electrode is 1.9 cm, and they are 8 cm in diameter. A position adjustable Zyvex S100 NanoEffecter probe is used to oscillate Melamine Formeldahyde (MF) particles levitated in a vertical chain in an open-ended glass box flush with the lower electrode. The chains are formed by dropping particles from a dust dropper at approximately 10 W rf power, which is then slowly decreased [2], [3]. The spherical particles have diameter 8.89  $\mu$ m and density 1.514 g/cm<sup>3</sup>. The glass box is 12 mm tall with equivalent walls that are separated by 10.5 mm and 2 mm thick. The probe tip is 50 nm in radius, attached above the upper electrode, and positioned vertically [2], [3].

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## II. METHOD

- A. Plasma Parameters
- 1) Resonance Frequencies

A chain is formed and all the particles are removed by adjusting the power until only one remains. The probe is positioned inside the box in a corner. The particle motion is observed and recorded by a camera above the box, in which the particle levitates near the center and moves horizontally away from the probe. The experimental parameters are: 2.22 W system power, 80 mTorr neutral gas pressure, -24 V natural DC bias, and 0 V probe bias. The probe drives the particle by oscillating the potential with a square wave at 1 Hz with amplitudes going from 15 to 70 V peak-to-peak. The particle is pushed by the wave and then oscillates around an equilibrium point before getting pushed again. The particle motion is recorded at 60 frames/second, and an underdamped harmonic oscillator equation [3] is fit to the particle's natural oscillatory motion. Resonance frequencies are then calculated from using the fitted damping coefficients and frequencies (1) and averaged together.

$$\omega_{\rm p} = \left(\omega^2 + \beta^2\right)^{\frac{1}{2}} \tag{1}$$

where  $\omega_0$  = resonance frequency,  $\beta$  = damping coefficient, and  $\omega$  = the frequency from the fitted equation.

2) DC Bias

The chain height above the lower electrode was measured for varying DC Biases at 2.42 W and 80 mTorr with a probe bias of 0 V and waves of amplitude 30 V peak-to-peak and frequency of 7 Hz.

### 3) Sheath Edge

Following Samarian's technique [4], the sheath edge is found by observing where a single MF particle, 0.46  $\mu$ m in diameter, levitates in the plasma for various powers at 80 mTorr with a natural DC bias of -25.5 V.

#### B. Dispersion Relations

The dispersion relations are found by fitting sine waves to the transverse motion of two particles in a chain at various frequencies and using the fitted phase differences to find the wavenumbers.

1) The probe is placed outside the box and moved down so that the tip is 1.51 mm down from the box's top and 2.74 mm from the box's wall. The parameters are: 2.14 W system power, 0.00421 W reverse power, -21.5 V natural DC bias, 79

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mTorr neutral gas pressure, and 0 V probe bias. A 4-particle chain is formed in the box and driven by a 50 V peak-to-peak sinusoidal probe potential in frequencies from 1 Hz to 10 Hz in order to create a transverse wave through the chain. The frequencies from 4 Hz to 10 Hz as well as the average distance of the top particle from the third particle down from the box's top are used to calculate a dispersion relation. Nearest neighbors are not used because the phase difference is too small.

The theoretical dispersion curve is modified from [5] to suit a vertical chain. The dispersion relation is,

$$\omega(k) = (\omega_0^2 - \frac{4Q^2}{4\pi \epsilon_0 M^2} \exp(-\frac{r}{\lambda_0})(1 + \frac{r}{\lambda_0}) \sin^2(\frac{kr}{\epsilon}))^{\frac{1}{2}}$$
(2)

where M is the dust mass (5.52 x  $10^{-13}$  kg),  $\lambda_D$  is the Debye length (700 microns [6]),  $\omega_0$  is the resonance frequency (50.8 rad/s to fit the data), k is the wavenumber, r is the mean distance between the particles throughout all frequencies (6.956 x  $10^{-4}$  m), and Q is the charge on all three particles (50590e).

2) The probe tip is placed 4.79 mm above the box and is put 0.77061 mm from the box's wall. The parameters are: 2.16 W system power, 0.00134 W reverse power, -19.9 V natural DC bias, 81 mTorr neutral gas pressure, and 0 V probe bias. A 5-particle chain is formed and transverse waves are driven through the chain by a sine wave of 40 V amplitude at frequencies of 5.0 Hz to 7.5 Hz. These frequencies and the average distance between the top particle and the next one underneath it are used to calculate a dispersion relation. The second particle from the top was more accurate in finding the phase difference because the lower particles had less exact sinusoidal motion. These parameters produced an excellent transverse wave through the chain.

## C. Model

A model is created to find the balance of forces and potential energy required to hold the particles in a 5-particle chain in the positions found from an experiment in which transverse waves were driven through that chain with 40 V amplitude sine waves at 7 Hz. In the experiment, the probe tip is placed at 5.01 mm above the box and either 0.915 mm, 1.55 mm or some mm in between from the box's wall. The parameters are: 2.42 W system power, 0.001805 W reverse power, -24 V fixed DC bias, 80 mTorr neutral gas pressure, and 0 V probe bias. The average heights of the particles above the lower electrode are calculated and used in the model. The model only deals with forces and potentials in the vertical direction. The following steps were conducted:

1) Using the method described earlier, the sheath edge is found at the parameters in which the chain was formed. It is 11.08 mm above the lower electrode.

2) The potential from the lower electrode is set to 0 V at the sheath edge and so the electric field is as well. By subtracting the plasma potential at 80 mTorr of 39.5 V from -24 V of the DC bias, -63.5 V is set at the lower electrode. The equation for the potential is

$$V(s) = \frac{V_{b} - V_{p}}{d^{2}} z^{4} + 2 \frac{V_{b} - V_{b}}{d} s + (V_{0} - V_{p})$$
(3)

where  $V_0 - V_p = -63.5$  V, d (sheath edge height) = 11.08 x 10<sup>-3</sup> m, and z is the height of the particle above the lower electrode. The interparticle interaction uses the nearest neighbor approximation and the Yukawa potential. The forces are found from the derivatives of the potential energy.

3) The gravitational potential and force is determined from the particle mass.

4) The charges are found, which sets the potential energy so that the middle particle in the chain is at the position of minimum total potential energy, and the forces are balanced so that the net force on each particle is zero at the corresponding particle's position.

#### III. RESULTS

#### A. Plasma Parameters

## 1) Resonance Frequency

The average resonance frequency was found to be 40.19 rad/s (6.40 Hz). The average damping coefficient  $\beta$  (beta) was 8.79 Hz. The results of the underdamped fit are shown in Figures 1-4. Fig. 1 shows and fits lines to the damping coefficient, the frequency from the fitted equation (damped frequency), and the resonance frequency. Fig. 2 shows the particle peak-to-peak amplitude and a fit to the points following a linear trend. The dashed line separates the outliers. The peak-to-peak amplitude was measured between the first crest and trough in the underdamped motion for each probe amplitude. Fig. 3 shows the particle distance from the probe's tip. Fig. 4 shows one of the fitted underdamped curves.



Fig. 1. Frequency and Beta from the fit, and the resonance frequency plotted against the probe's peak-to-peak amplitude.

Amplitude from Damped Fit 1.9 1.8 1.7 0.6 0.5 0.4 0.3 0.2 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 Probe Peak-to-Peak (V)

Fig. 2. The particle peak-to-peak amplitude plotted against the probe peak-to-peak amplitude applied to the particle.





Fig. 4. One of the underdamped fitted curves.







## B. Dispersion Relations

1) In Fig. 7, the data points start at the arrow labeled "start" and follow the arrows. The blue quadratic fits the first four points. The green curve is the theoretically predicted curve that fits the next two points. The red line connects the last three points. The line and the quadratic are there to suppose different wave motion requiring different dispersion theory.



Fig. 7. The dispersion relation for the 4-particle chain. The data points start at the arrow labeled "start" and follow the arrows. The green curve is the theoretically predicted curve. The blue quadratic fits the first four points. The red line fits the last three points.

2) No theoretical curve has been fit for Fig. 8 yet.

## C. Model

The results of the model are shown in Figures 9 and 10: where the light blue lines near 0 are the net particle interaction, the dark green line in the potential energy graph (Fig. 9) and constant in the force graph (Fig. 10) is the contribution from gravity, the black lines are the electric field from the lower electrode, and the red parabola in Fig. 9 and the red lines in Fig. 10 that intersect 0 where a particle intersects 0 are the sum of all these contributions. The five blue dashed lines are the positions of the particles in the chain. The vertical dark red line in Fig. 9 marks the minimum of the total potential energy and so the minimum of the red parabola.

A Debye length of 700 microns was used for each particle. However, the graphs are fairly independent of the Debye length used. The net interparticle repulsion moves very little with a large change in Debye length in either direction. The required charges on the particles from top to bottom were -37500e, -24700e, -18400e, -14200e, -11600e, where *e* is the charge of an electron. The net interparticle interaction makes a much smaller contribution to both the total potential energy and net force than gravity or the electric field.



Fig. 9. The potential energy from the model. The dark green is from gravity, the light blue is from interparticle repulsion, the black is from the electric field, the red parabola is the total potential energy, the dashed dark blue lines are the particle positions, and the dark red intersecting the parabola is the minimum total potential energy.



Fig. 10. The forces from the model. The dark green is from gravity, the light blue is from interparticle repulsion, the black is from the electric field, the red lines intersecting zero at particle positions is the net force, and the dashed dark blue lines are the particle positions.

#### IV. DISCUSSION

## A. Plasma Parameters

## 1) Resonance Frequency

The frequencies and resonance frequencies resulting from the fitted equations are nearly constant (Fig. 1). The average resonance frequency makes sense because from around 5 to 7 Hz is where the best transverse waves through the chain were found. The average value of 8.79 Hz for  $\beta$  is close to the expected value of 8 Hz. However,  $\beta$  shows some decrease with increasing amplitude, although it is expected to be constant. This could be due to a smaller camera resolution that did not capture the more quickly moving particle at those probe amplitudes. However, the graph of the particle oscillation amplitude (Fig. 2) suggests that at higher amplitudes the assumption of underdamped oscillation no longer holds because the particle oscillation amplitude no longer increases with increasing probe amplitude. The probe at high potentials is probably changing the conditions by absorbing many electrons. There could also be higher order terms at those amplitudes from the horizontal confinement. Ignoring that data gives an average  $\beta$  of 9.22 Hz, which is higher than expected but close enough to 8 that this could be within experimental error. Because the resonance frequency hardly changed with increasing amplitude, the value for it can be trusted even at high peak-to-peak probe amplitudes.

## 2) DC Bias

Fig. 5 shows that negative DC biases move the chain upwards while positive DC biases move the chain downwards—just as expected. The bump in the data that occurs where the particle loses a chain probably happened because the data was taken at -24 V and then moved to -30 V. At -30 V, arcing occurred and a particle was lost, so the plasma conditions probably were different for the last measurements up to 10 V. The large data gap could be filled in future experiments so that a trend can better be seen.

## 3) Sheath Edge

The sheath edge moves up, reaches a maximum, and then decreases to some equilibrium value as power increases (Fig. 6). A fourth order polynomial is the smallest degree that fits the sheath edge plot. The predicted sheath edge for the power value used in the model found when varying the power is less than the 11 mm used in the model found at another time. Whether the sheath edge plot given is accurate quantitatively could be determined by future experiments.

## B. Dispersion Relations

1) The theoretical curve predicts a resonance frequency greater than the one found (40.19 rad/s) by the underdamped fitting to square wave oscillations. Taking data at a higher frame rate could produce a higher resolution to which to fit the curve. The theoretical curve also does not take into account density, charge, Debye length, and electron temperature changing in the vertical direction. These could lower the predicted curve enough that it matches with the earlier found resonance frequency.

Future work could find fits for the other parts of the dispersion relation. The variance in the dispersion relation suggests that there are three different wave motions going on through the chain as the frequency changes. Also, the imaginary parts of the dispersion relation could be examined.

2) The dispersion relation for the 5-particle chain (Fig. 8) is similar to that of the 4-particle chain for lower frequencies, which confirms the presence of this trend.

## C. Model

The model confirms the decreasing of charge in the particles of the chain as they levitate closer to the lower electrode. However, the Debye length being able to change so much suggests additional restraints are required in order to find an accurate Debye length—one that changes with height. One possibility includes the possibility of vertical forces from the box walls.

The small net interparticle interaction compared to the electric field and gravity, and changing so little with Debye length, means that it contributes little in determining where the particles levitate in the box.

## V. CONCLUSION

The measurements of the resonant frequencies, damping coefficients, and amplitude of particle oscillation show that the resonant frequency stays mostly constant despite the probe amplitude increasing so much that the underdamped fit to the particle oscillation no longer holds (> 40 V). The damping coefficient changes with the amplitude, but if the data above 40 V peak-to-peak amplitude on the probe is ignored, the average damping coefficient is larger than expected.

The DC bias moved the chain as expected—negative upwards, and positive downwards. The sheath edge changed with power and could be fit by a fourth order polynomial, but more work could be done to check the quantitative accuracy of these measurements.

The dispersion relations show that there are at least three kinds of wave motion going through the chain, and the theoretical curve fit the transverse motion after it started damping with increased frequency. The theoretical curve could be greatly improved with more data and by taking into account more variables, and theoretical curves still need to be found for the other wave motions.

The model shows that the net interparticle interaction has relatively little effect in determining where the particles levitate in the box: the electric field and gravity mainly determine that. The model confirmed the decrease of charge from top to bottom in a vertical chain, and it worked with an expected Debye length. However, the model changes little with different choices of Debye length and so could be improved, including by taking into account possible vertical forces from the box edges.

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