Systematically Generated Layer 1 Weakly Coupled Free Fermionic Heterotic String Models

Rachel Elliott*, Jared Greenwald, Doug Moore, Brandon Mattingly and Gerald Cleaver
CASPER, Department of Physics, Baylor University, Waco, TX, 76798-7316, USA

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***Abstract*—Layer 1 weakly coupled free fermionic heterotic string models of orders 1-5 were systematically generated. Among these models 36 contained the following grand unification theory groups: 1 Pati-Salam, 3 E_6 and 32 SO(10). The number of spacetime supersymmetries, N, of the models were also found with $N \in \{0, 2, 4\}$ for even ordered layer 1 models and $N \in \{0, 4\}$ for odd ordered layer 1 models. Finally, the general layer 1 models were compared with gauge models.**

***Index Terms*—String Theory, Systematic Model Building, Low Energy Effective Field Theory.**

I. Introduction

STRING theory is not yet a single theory in itself but an umbrella term for theoretical models using strings as the most fundamental matter. There are postulated to be on the order of 10^{500}different configurations within the String Theory Landscape [1][2][10].

Other theories are capable of providing quantum theories of three of the fundamental forces of nature, namely the strong, weak and electromagnetic force, but lack the necessary tools for a reasonable quantum theory of gravity. In string theories not only can quantum theories of the strong, weak and electromagnetic forces be found, but the graviton (the quantum of the gravitational field) appears as a vibrational mode of closed strings; therefore allowing for a quantum theory of all four of the fundamental forces. This opens up the possibility of providing a single unique unified theory of all fundamental interactions which string theorists strive to find [3].

The weakly coupled free fermionic heterotic string (WCFFHS) formalism has been successful in generating promising phenomenological models [15-24]. This mathematical structure is also convenient for building systematic, computer automated models [4]. Therefore this area of string theory is of interest to study further.

A. Layer 1 WCFFHS Model Building

The WCFFHS formalism has a radius of theory, R, such that R =1 with a vanishing coupling constant g << 1, and is built upon a 2-torus world sheet with non-interacting fermions. Left movers are the modes associated with the 10-dimensional (half-integer) fermionic superstrings whose vibrational modes describe matter. Right movers are the modes associated with (integer) bosonic strings propagating in 26-dimensions whose vibrational modes describe the force carriers. The WCFFHS structure consists of closed strings and thus has disjoint sets of left and right moving modes.

Two inputs are specified for constructing layer 1 WCFFHS models, namely the basis vector set and the GSO coefficient matrix. A number of modular invariance constraints are imposed on these inputs in order to ensure quantum mechanical consistencies. The basis vector set fully defines the compactification of space by specifying the phase gained by parallel transporting fermion modes around non-contractible loops on the worldsheet [4].

Basis vector set is defined as:

$$\boldsymbol{A} = \left\{\vec{\alpha}_i \middle| \vec{\alpha}_i \ \in \mathbb{Q}^{64} \cap (-1,1]^{64}\right\} \tag{1}$$

For layer 1 models the basis vector sets consist of the all-periodic $\vec{\alpha}^B_{\mathbb{1}} = \langle 1^{20} | 1^{44} \rangle$, the SUSY basis vector $\vec{\alpha}^B_{\vec{S}} = \langle 11(100)^6 | 0^{44} \rangle$ and one additional

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*R. Elliott is a Baylor University REU fellow with Virginia Tech, Blacksburg, VA 24061 USA (phone: 410-340-2918; e-mail: rachelelliott13@ gmail.com).

Dr. G. Cleaver is with Baylor University, Waco, TX 76798 USA (e-mail: gerald_cleaver@baylor.edu).

D. Moore is with Baylor University, Waco, TX 76798 USA (e-mail: Douglas_Moore1@baylor.edu).

J. Greenwald is with Baylor University, Waco, TX 76798 USA (e-mail: Jared_Greenwald@baylor.edu).

B. Mattingly is with Baylor University, Waco, TX 76798 USA (e-mail: Brandon_Mattingly@baylor.edu).

basis vector. The number of possible values of phases for the additional basis vector, called the order, N, of that basis vector also needs to be specified.

Sectors $\vec{\alpha}$ are built by taking all linear combinations of basis vectors in the inputted set and from these massless states are built. After projecting away any unphysical states using the inputted GSO coefficient matrix gauge groups and the number of spacetime supersymmetries are found and from these the force and matter content is determined [4].

1) Modular Invariance Constraints

Letting N_i be the order of basis vector α_i^B and $N_{ij} = lcm(N_i, N_j)$. The following modular invariance constraints must be met by the inputted basis vector in order to ensure quantum mechanical consistency by ensuring the physics remains the same if your parameters are rotated 360 degrees along a non-contractible loop on the world sheet:

$$N_i \alpha_i^{B\,2} \equiv 0 \; mod \; 8 \quad \text{if } N_i \text{ even} \tag{2}$$

$$N_i \alpha_i^{B\,2} \equiv 0 \; mod \; 4 \quad \text{if } N_i \text{ odd} \tag{3}$$

$$N_{ij} \alpha_i{}^B \cdot \alpha_j{}^B \equiv 0 \; mod \; 4 \tag{4}$$

The second input, namely the GSO coefficient matrix with elements k_{ij} used in GSO projections in one of the final steps of the model building process has the following constraints:

$$N_j k_{ij} \equiv 0 \; (mod\; 2) \tag{5}$$

$$k_{ij} + k_{ji} \equiv \frac{1}{2} \vec{\alpha}_i{}^B \cdot \vec{\alpha}_j{}^B \; (mod\; 2) \tag{6}$$

$$k_{ii} + k_{j1} \equiv \frac{1}{4} \vec{\alpha}^B{}_i{}^2 - s_i \; (mod\; 2) \tag{7}$$

Where s_j is 0 for bosonic basis vectors and 1 for fermionic basis vectors.

2) Building Sectors and Massless States

The geometry of the model is built by taking linear combinations of the basis vectors, these form the sectors $\{\vec{\alpha}\}$. Sector describe the transformation of fermions around non-contractible loops on the world sheet [10].

Letting m_{ij} be an integer constrained by $m_{ij} = \{0, 1, \dots, N_j - 1\}$, sectors are constructed using

$$\vec{\alpha}_i = \sum_i m_{ij} \vec{\alpha}_j^B \tag{8}$$

Massless states are built using the relation:

$$\vec{Q} = \frac{1}{2} \vec{\alpha} + \vec{F} \tag{9}$$

Where $\vec{F}$, the fermion number operator [10], is a vector of all combinations of $0, \pm 1$ such that the state $\vec{Q}$ is massless according to the conditions:

$$\vec{Q}_L^2 = 2 \tag{10a}$$

$$\vec{Q}_R^2 = 4 \tag{10b}$$

3) GSO Projection

Let $\vec{Q}_{\vec{\alpha}}$ be the state generated from sector $\vec{\alpha}$, m_i be the coefficients on the basis vector set used to construct $\vec{\alpha}$ and s_j be 0 for bosonic basis vectors and 1 for fermionic basis vectors [3]. In the WCFFHS formalism the GSO projection is given by:

$$\vec{\alpha}^B{}_i \cdot \vec{Q}_{\vec{\alpha}} \equiv \sum_j a_j k_{ij} + s_j \; (mod\; 2) \tag{11}$$

The GSO Projection removes all unphysical states such as the state associated with tachyons.

B. Grand Unification Theory Groups

Grand unification theories (GUTs) are theories in which at sufficiently high energies the strong nuclear, weak nuclear and electromagnetic forces are joined into one unified force [1]. GUT groups, which are gauge group products associated with GUTs include the flipped SU(5)⊗U(1), SO(10) [5][6], SU(4)⊗SU(2)⊗SU(2) (Pati-Salam) [5], SU(3)⊗SU(2)⊗SU(2) (left-right symmetric) [7], E_6, and the well-known 1-2-3 symmetry group of the standard model: SU(3)⊗SU(2)⊗U(1) [6]. Models containing these gauge group products as a subgroup are more likely to have phenomenologically realistic results when further developed.

C. Spacetime Supersymmetries

The number of spacetime supersymmetries (ST SUSYs), $\boldsymbol{N}$ specifies the number of distinct copies of supersymmetry generators [7]. Thus the number of ST SUSYs distinguishes the number of fermion superpartners for each boson and the number of boson superpartners for each fermion.

The number of ST SUSYs can be found by counting the number of gravitinos. On our worldsheet gravitinos are of the form:

$$\left\langle \frac{1}{2}\frac{1}{2}\left(\pm \frac{1}{2}\; 00\right)^6 \middle| 0^{44} \right\rangle \tag{12}$$

Since every element in the real basis vector has to form a pair with another element the 64 possible gravitinos narrow down to 8 possibilities [8]. As is clear from their form, gravitinos come from the sector generated by the

SUSY basis vector. Therefore, in the GSO projection, the dot product on the left-hand side of equation 11 will be between the gravitino state vector and the SUSY sector. This also means that the coefficient a_j on the right-hand side of equation 11 will be 0 for $j = 1$ and $j = 3$ and 1 for $j = 2$. Since the elements of the coefficient matrix are either 0 or 1, this implies that the dot product between the gravitino state vector and the SUSY sector must either be congruent to 0 or 1 mod 2.

Certain values of $\boldsymbol{N}$ are more likely to produce results that are more phenomenologically realistic. Also some values of $\boldsymbol{N}$ require more arduous calculations in order to find values such as the vacuum expectation values and the results of SUSY breaking. Therefore $\boldsymbol{N}$ values for each model are of interest for us to find.

D. *Gauge Models*

A gauge model is defined as follows:

> A model is a **gauge model** if it can be built from a set of basis vector in which every basis vector beyond the all-periodic and SUSY basis vector is bosonic, that is of the form $\langle 0^{10} | \vec{\alpha}^{22} \rangle$, within the free fermionic construction. [10-14]

Note that the number of elements in the basis vector is halved due to using a complex basis. Gauge models have the potential to develop a new perspective for systematic model building where the gauge model is used as a foundation to build mode intricate models upon [10]. Since the additional basis vector used to construct gauge models has more constraints it is reasonable to expect every gauge model to be found within the set of general models.

II. Results

A. *Statistics for Layer 1 Models*

Statistics were found for all unique layer 1 models of order 1-5 in order to determine which models are likely to be phenomenologically realistic and therefore of interest for further development.

> A model is considered unique if no other model has been previously generated with both the same gauge group and number of space-time supersymmetries [10].

1) *Order 2*

Models were generated using an additional basis vector of order 2. There were 36 unique models found among 145320 consistent models as seen in Table I.

TABLE I

Number of Basis Vector Sets Tested, Consistent Basis Vector Sets, Consistent Models and Unique Models For Orders 1-5

Order	BVS Tested	Consistent BVS	Consistent Models	Number of Unique Models
2	290304	36332	145320	36
3	290304	12096	47376	12
4	29030400	3619328	13896576	199
5	29030400	725760	2857428	36

SO(10) was the only GUT group found among order 2 layer 1 models. SO(10) was found in 3 unique models as shown in Table II.

TABLE II

GUT Groups Found for Layer 1 Order 1-5

Order 2

GUT Group	Number of Unique Models	% of Unique Models
$SO(10)$	3	8.333%

Order 3: No GUT Groups

Order 4

GUT Group	Number of Unique Models	% of Unique Models
E_6	1	0.5025%
$SO(10)$	25	12.56%
$SU(4) \otimes SU(2) \otimes SU(2)$	1	0.5025%

Order 5

GUT Group	Number of Unique Models	% of Unique Models
E_6	2	5.556%
$SO(10)$	4	11.11%

The number of ST SUSYs were also found for each unique model. There were 15 unique models with $\boldsymbol{N} = 0$ ST SUSYs, 8 unique models with $\boldsymbol{N} = 2$ ST SUSYs and 13 unique models with $\boldsymbol{N} = 4$ ST SUSYs as seen in Fig 1.

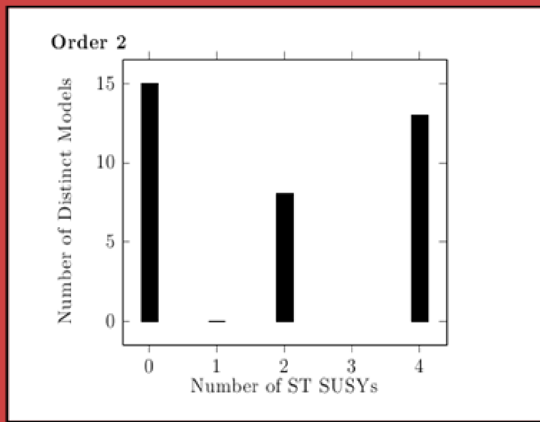

Fig. 1. Histogram of the frequency of different number of spacetime supersymmetries for order 2 models

2) Order 3

Models were generated using an additional basis vector of order 3. There were 12 unique models found among 47376 consistent models as seen in Table I.

No GUT groups were found among order 3 layer 1 models.

The number of ST SUSYs were also found for each unique model. There were 6 unique models with $\boldsymbol{N} = 0$ ST SUSYs and 6 unique models with $\boldsymbol{N} = 4$ ST SUSYs as seen in Fig 2.

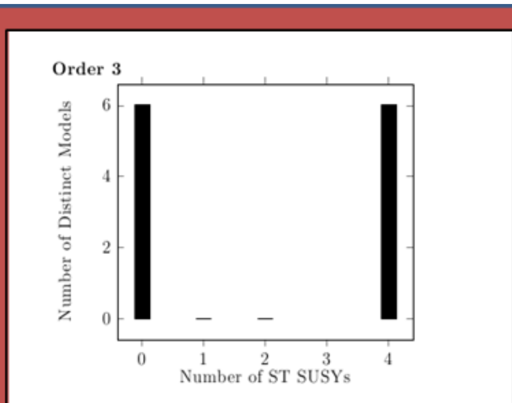

Fig. 2. Histogram of the frequency of different number of spacetime supersymmetries for order 3 models

3) Order 4

Models were generated using an additional basis vector of order 4. There were 199 unique models found among 13896576 consistent models as seen in Table I.

E_6, SO(10) and the Pati-Salam GUT groups were found among order 4 layer 1 models. SO(10) was found in 25 unique models while only one E_6 and Pati-Salam GUT group were found as shown in Table II.

The number of ST SUSYs were also found for each unique model. There were 118 unique models with $\boldsymbol{N} = 0$ ST SUSYs, 35 unique models with $\boldsymbol{N} = 2$ ST SUSYs and 46 unique models with $\boldsymbol{N} = 4$ ST SUSYs as seen in Fig 3.

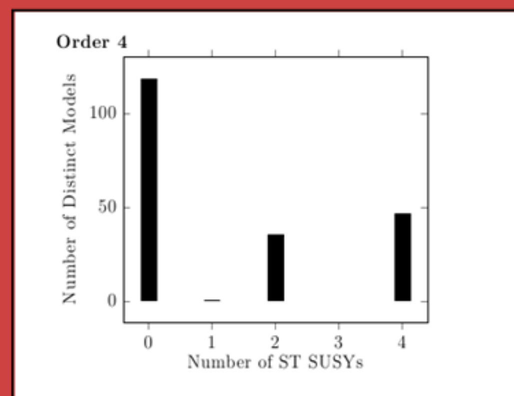

Fig. 3. Histogram of the frequency of different number of spacetime supersymmetries for order 4 models

4) Order 5

Models were generated using an additional basis vector of order 5. There were 36 unique models found among 2857428 consistent models as seen in Table I.

E_6 and SO(10) GUT groups were found among order 5 layer 1 models. SO(10) was found in 4 unique models and E_6 was found in 2 as shown in Table II.

The number of ST SUSYs were also found for each unique model. There were 18 unique models with $\boldsymbol{N} = 0$ ST SUSYs and 18 unique models with $\boldsymbol{N} = 4$ ST SUSYs as seen in Fig 4.

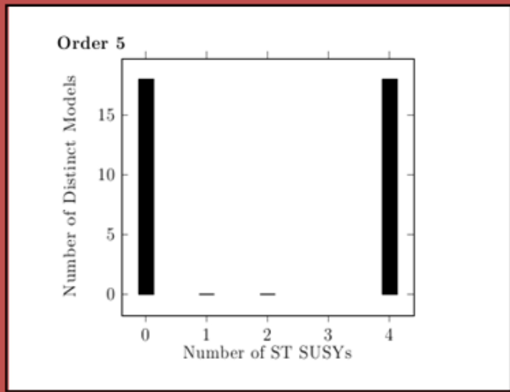

Fig. 4. Histogram of the frequency of different number of spacetime supersymmetries for order 5 models

III. Conclusion

Among 283 unique layer 1 models generated for orders 1-5, 36 contained GUT groups. There were 1 Pati-Salam, 3 E_6 and 32 SO(10) found in total. This leaves only 36 out of 283 models as phenomenologically interesting and therefore only these 36 will be developed further. In future work vacuum expectation values and the results of SUSY breaking will be found for a fully developed model.

The number of ST SUSYs will be used in these computations. It has been found that the dot product between the gravitino state vector and the SUSY sector must either be congruent to 0 or 1 mod 2 due to the GSO projection shown in equation 11. Therefore it is reasonable that no layer 1 models were found to have $\boldsymbol{N} = 1$ or $\boldsymbol{N} = 3$ ST SUSYs. It is also due to this constrain placed on the gravitinos that both sets of odd ordered layer 1 models contained no models with $\boldsymbol{N} = 2$ ST SUSYs because any additional basis vector that met the modular invariance constraints produced gravitinos which were removed by the GSO projection.

The general models were compared to the gauge models which were built using different methods placed on them for efficiency. This comparison revealed a flaw in the gauge model building program which, when trying to remove redundant models was removing models that contained the same physics as another while having different gauge groups. For example among the following models (note that the gauge models are built using a complex basis)

$$\langle 0^{10} | 11110^{18} \rangle \tag{13a}$$

$$\langle 0^{8} 11 | 110^{20} \rangle \tag{13b}$$

Equation 13b would be removed as being a redundancy of equation 13a, generating an incomplete set of gauge models.

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