

Paul Hagelstein

Mathematics / College of Arts & Sciences

Harmonic Analysis: Problems in Multiscale Differentiation Theory

This proposal concerns fundamental problems related to the theory of differentiation of integrals and strategies for attacking them utilizing mathematical techniques developed in the past decade. The theory of differentiation of integrals occupies a venerable role in analysis, having first come to light with the first rigorous proof of the Fundamental Theorem of Calculus and subsequently finding far-reaching applications in fields ranging from several complex variables to partial differential equations. The theory of differentiation of integrals achieved an especially high level of prominence in analysis when it was used to solve a wide array of problems involving regularity properties of singular integral operators, multiplier operators, and the convergence of Fourier series. In spite of its importance, however, differentiation theory as a whole has been an unusually slow moving field over the past thirty years. The reason is not surprising, as in the late 1970's the natural development of the field encountered seemingly intractable fundamental problems associated to issues surrounding the halo conjecture and placing sharp Orlicz weak type bounds on a host of geometric maximal operators ranging from the lacunary maximal operator to maximal operators associated to hypersurfaces in space.

That being said, over this past decade analysts such as Bourgain, Katz, and Tao have encountered considerable success in using probabilistic techniques to yield impressive results regarding multilinear operators and lower bounds on the dimension of Besicovitch sets. The viability of these combinatorial techniques in the harmonic analysis setting suggests that some older and previously-considered intractable problems in the theory of differentiation of integrals might now be approached using probabilistic methods with a reasonable chance of success. The goal of this proposal is to determine if key model unsolved problems related to the theory of differentiation of integrals may be solved by these combinatorial and probabilistic techniques. Problems of particular interest involve the halo conjecture, sharp relations between covering lemmas and maximal operators of weak type, and multiscale covering lemmas. These model problems have been carefully selected to be accessible to a highly motivated mathematically talented undergraduate and also provide an effective gateway to more advanced research in harmonic analysis.