Normal mode couplings for three- and ten-microparticle systems suspended in plasma

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Abstract

A box_tree code was used to simulate the motion of three- and ten- particle plasma crystals. The ion wake effect was included in the simulations in an effort to determine if it affected small plasma crystals in the same way that it affects large ones. It was found that the ion wake effect lead to mode couplings, and the mode couplings often lead to resonances when two coupled modes were near the same frequency. These resonances often caused instabilities in the crystal. In addition, it was often observed that when resonance occured, many modes that were not coupled to the resonating modes would vibrate at the resonant frequency. These results are very similar to the couplings and instabilities presented in [1] and [2] and the non-dispersive mode presented in [1] for large plasma crystals. In addition, the mode couplings once caused a frequency gap for two modes, the theoretical basis for which was established in [3].

1 Introduction

Complex (or dusty) plasmas are important in many areas of research, including astrophysics, where they are found in planetary rings and play a role in protoplanet formation [4]. They are also found in the fabrication of microelectronics, where dust particles can be suspended in the plama, then fall and contaminate the surface when the plasma is no longer there [5]. In experiments performed on earth, complex plasmas are often studied by immersing dust in a weakly ionized gas. Thus, there are four components to complex plasmas: electrons, ions, neutral gas atoms, and dust particles. The dust particles acquire electric charge via collisions with electrons and ions and their motion is damped by the neutral atoms. Since the electrons are much more mobile than the ions, the dust particles are usually charged negatively. An electrode levitates the dust particles by either producing a constant potential at its surface or by producing an AC potential with a negative bias. If the potential at the electrode's surface alternates, it is varied too quickly (usually at radio fequencies) to have a measurable effect on the dust particles. Thus, the particles are confined vertically by gravitational and electrostatic forces. To confine the particles horizontally, the electrode can have a cutout or a metal ring placed upon it. The cutout or ring produces an electric field with a radial component that is approximated as a linear function of distance from the center of the electrode [6]. In this way, the particles are horizontally confined by the electric field due to the cutout or ring [7]. Alternatively, a glass box can be used to confine the particles horizontally [8].

In addition, the interparticle interactions are screened by the surrounding ions [9]. Thus, instead of a Coulomb interaction between two particles, where the potential φ is inversely proportional to the distance r that separates the particles, a Yukawa potential, which has the form $\varphi \propto (1/r) \exp(-r/\lambda)$, is used [7]. The length λ is called the Debye length.

Under the conditions described above, the dust particles will self-organize into a plasma crystal, which has a hexagonal structure [5]. Since their discovery in 1994, plasma crystals have been thoroughly studied because they are in many ways analogous to solid state systems. In particular, waves that travel through plasma crystals are often studied [10]-[12] because they indicate how the particles interact with each other [10].

The study of these waves has proven to be complex due to the ion wakefield. The ion wake effect is the focusing of ions below the dust particles. This happens because the positively charged ions move downward toward the negatively charged electrode. As they approach the negatively charged dust particles, they are attracted to them, and their paths are deflected [7]. The ion wakefield may explain couplings that have been observed between vertical and horizontal oscillations [3], [1], [6].

These mode couplings have been studied both theoretically [3] and experimentally [1], [2]. When considering crystals of large spacial extent, dispersion relations are generally used to study the waves that travel through them [1], [2], [3], [11]. Here, however, finite crystals consisting of three and ten particles were studied in an attempt to determine if the mode couplings are present in small systems of dust particles. In this case, it is better to use mode spectra to study the oscillations [10], [12], [6].

2 Method

A numerical simulation was performed using the box_tree code [13]. Box_tree allows the user to input boundary and initial conditions; the number, sizes, charges, and masses of dust particles; the Debye length; gas drag coefficients; and the horizontal and vertical confinements. As noted above, the projection of the electric field onto the x-y plane is oriented radially outward and is approximated as a linear function of distance from the origin, i.e. [6]

$$E_{\rho} = E_{\rho}^{\prime} \rho. \tag{1}$$

The vertical component of the electric field is also approximated in this way [6]:

$$E_z = E_0 + E'_z z. (2)$$

Thus, the values for the confinements, called confinement coefficients, are derivatives of the electric field in V/m^2 .

In the simulations presented here, the dust particle systems were unbounded and initially randomly distributed. The particles had a constant charge of -3 fC, a radius of 4.71 μ m, and a density of 1.51 g/cm³, resulting in a mass of $6.61 \cdot 10^{-13}$ kg. The Debye length was .5 mm. The gas drag coefficients were set to 10 s^{-1} for the first 15 s of the simulations, then set to 0 s^{-1} afterward. This damped the motion of the particles at first, causing them to organize into crystals quickly. The undamped motion that followed facilitated the observations of the particles' oscillations after they had attained a crystal structure. The value of the vertical confinement coefficient was kept at $2.16 \cdot 10^6 \text{ V/m}^2$ for all simulations. However, the values of the confinement coefficients in the x and y directions, while always equal to each other, varied between simulations and ranged from $1 \cdot 10^5 \text{ V/m}^2$ to $1 \cdot 10^6 \text{ V/m}^2$.

Box_tree approximates the ion wakefield by placing an imaginary positively-charged particle under each dust particle [6], [14]. Thus, while a dust particle is repelled by the neighboring dust particles, it is attracted to the neighboring imaginary particles. The position and charge of the particle is input by the user. In these simulations, the imaginary particles were placed .25 mm, or half of the Debye length, below the dust particles. For the simulations involving three particles, the imaginary particle's charge was .6 fC, or one-fifth of the magnitude of the dust particle charge. For reasons explained in the next section, however, the imaginary particle charge was reduced to .3 fC when ten-particle systems were studied.

During the simulations, box_tree calculated the forces on each particle and used them to determine each particle's position and velocity for the next time step. The time interval between each step depended on the magnitudes of the forces on each particle. The program then created data files at user-defined intervals of .02 s that contained the positions and velocities of each particle. The program created 994 such data files, meaning the particles' oscillations were observed for a total of 19.88 s.

After the simulations were run, a MATLAB code was used to first calculate and then plot the average positions of each particle. This made it easy to determine if the particles had indeed attained a crystal structure or if the crystal was unstable for that run; if the structure was unstable, the average positions would be randomly distributed.

Beyond this initial first step to obtain the mode spectra, the data analysis was done differently for the three-particle systems than for the ten-particle systems. For the three-particle systems, a Fourier transform was applied to each component of the velocity data, giving the frequencies at which the system vibrated in the x, y, and z directions and the motion of the system at each such frequency.



Figure 1: The four types of horizontal modes. The rotation mode is shown by the upper left image, a sloshing mode is shown by the upper right image, a kinking mode is shown by the lower left image, and a breathing mode is shown by the lower right image. Figure modified from [10].

For the ten-particle systems, MATLAB used the average positions and confinement coefficients to calculate the dynamic matrix of the system, without taking into account the ion wake effect. The eigenvalues and eigenvectors of the dynamic matrix are the frequencies and velocities, respectively, of the modes that would be present in the system without the ion wake effect [10], [6]. This allowed the actual motion of the particles to be studied in terms of the non-coupled modes and allowed for separation between the vertical and horizontal modes. The velocities from the simulation data were then projected onto the eigenvectors for each time step, giving the dot products of the actual velocities with the velocities of each mode as functions of time [10]:

$$v_l(t) = \sum_{i=1}^{N} \vec{v}_i(t) \cdot \vec{e}_{i,l},$$
(3)

where N is the total number of particles and $\vec{e}_{i,l}$ is the eigenvector of the ith particle in the l^{th} mode. Those functions of time were then Fourier transformed, giving the projections of the velocities as a function of frequency for every mode [10]:

$$S_{l}(\omega) = \frac{2}{T} \left| \int_{-T/2}^{T/2} v_{l}(t) e^{-i\omega t} dt \right|^{2},$$
(4)

where $S_l(\omega)$ is the spectral power density.

3 Results

3.1 Three-Particle Systems

For the three-particle systems, there are nine normal modes. Six modes are in the horizontal direction: one rotation mode, where the entire system of particles rotate; two horizontal sloshing modes, where the center of mass of the system oscillates; two kinking modes, where two particles pinch together and one moves away; and a breathing mode, where the entire system expands and contracts [10] (Figure 1). For the other three modes, the motion was exclusively in the vertical direction: two modes with relative vertical motion, where two particles oscillated up and down in phase and the other oscillated up and down 180 degrees out of phase with the other two; and the vertical sloshing mode [12].

Figure 2 shows the mode frequencies as functions of the horizontal confinement coefficient. The rotation and vertical sloshing modes are independent of the horizontal confinement coefficient. The other horizontal modes follow functions of the form $f = Ax^{\beta}$, where A and β are real numbers and $0 < \beta < 1$, very well. In fact, the R^2 value for these modes is never less than 0.9998 when rounded to four significant digits. The vertical relative mode's dependence on the horizontal confinement coefficient is reasonably approximated with a quadratic function.

Figure 3 shows Fourier transform data in the x and z directions. The spikes show at which frequencies the crystal vibrates. Additionally, the small spikes in the x(z) direction that coincide with larger spikes in the z(x) direction show which horizontal (vertical) modes are coupled with which vertical (horizontal)



Figure 2: A plot of all of the mode frequencies as a function of confinement coefficient for the three-particle system. The horizontal modes follow power laws well.

modes. It was determined from data like this that the horizontal sloshing and kinking modes are coupled with the vertical relative modes, the vertical relative modes are coupled with the kinking modes, and the vertical sloshing mode is coupled with the breathing mode.

These mode couplings affect the motion of the particles most strongly when the frequencies of the two modes are near each other. In that case, there are two things that can happen. Most of the time, the two modes will get close enough in frequency that resonance occurs. When there is resonance, almost all of the kinetic energy of the system goes into the at least one of the two modes, as shown by Figure 4. When the two modes are at the exact same frequency, the resonance will often cause the crystal to be unstable. The three-particle systems are unstable when the confinement coefficient has a value of $6.13 \cdot 10^5 \text{ V/m}^2$ due to the coupling between the vertical sloshing and breathing modes.

However, the coupling between the kinking and vertical relative modes causes a frequency gap for both modes between about 12.1 and 12.6 Hz that prevents the two modes from having the same frequency. Thus, no resonance occurs, and no instability results from this coupling (Figure 5).

3.2 Ten-Particle Systems

The ten-particle crystals had two different crystal structures, as shown in Figure 6. One of the crystal structures has three particles in the middle with the other particles surrounding them, while the other has two particles in the middle and four in a line. For each crystal structure, there were 30 modes: 20 horizontal and 10 vertical. Each crystal structure had different modes, and in addition, different degeneracies, as shown by Figure 7, which shows the spectral power density of each mode as a function of frequency.

With an imaginary particle charge of .6 fC, the crystals were unstable at a broad range of confinement coefficients. Therefore, in the interest of having the ability to gather more information about the modes, the imaginary particle charge was reduced to .3 fC. This reduced the strength of the coupling between the modes, and allowed for stable crystals for more values of the confinement coefficient. Once we had more simulations with stable crystals, it was much easier to distinguish which modes were coupled.

Once again, the rotation and vertical sloshing modes (mode numbers 1 and 30, respectively) are not affected by the confinement coefficient, the other horizontal modes (mode numbers 2 through 20) follow power laws, and the other vertical modes (mode numbers 21 through 29) could be approximated with quadratic functions.

Figure 8 shows the instabilities for each crystal structure, as well as the coupled modes that cause them.



Figure 3: The Fourier Transform of the x-components of the velocities (top) and the z-components of the velocities (bottom).



Figure 4: The Fourier Transform of the x-components of the velocities (top) and the z-components of the velocities (bottom) when the confinement coefficient was $6.13 \cdot 10^5 \text{ V/m}^2$. Since the vertical sloshing mode is coupled with the breathing mode and both modes are at the same frequency, the peak for the vertical sloshing mode, which has a dimensionless frequency of 314 in this image, is much higher than any other peak.



Figure 5: The coupling between the kinking and vertical relative modes caused a frequency gap, as shown by the fact that neither mode ever has a frequency between 12.1 and 12.6 Hz.



Figure 6: The ten-particle systems had two crystal structures, as shown here.



Figure 7: The colors correspond to the spectral power density, with red being a high value and dark blue being zero. Mode numbers are on the horizontal axis, while the frequency scaled by the total time of observation is on the vertical axis. Each crystal structure had different modes that vibrated at different frequencies.



Figure 8: Trendlines for some of the modes in each crystal structure, with red dots where there are instabilities and blue dots where there are resonances without instabilities. Crystal structure 1 has three particles in the center, and Crystal Structure 2 as two particles in the center.

If two modes are coupled but do not cause an instability when they are at the same frequency, there is a blue dot at their cross point. If two modes cause an instability, there is a red dot at their cross point.

Sometimes, when two modes resonated, most other modes would vibrate at the resonant frequency, as shown in Figure 3.2. This happened even though not all the modes that vibrated at the resonant frequency were coupled to the two resonating modes. This was observed for both crystal structures, but not for every resonance.

4 Discussion

Although it is unclear why all horizontal mode frequencies should have a functional dependence on the confinement coefficient in the form of a power law, the horizontal sloshing mode is theoretically predicted to have a frequency that is proportional to the square root of the confinement coefficient [7]. This agrees extremely well with the results obtained for the horizontal sloshing modes of the three-particle systems, which were proportional to the confinement coefficient raised to the power of .4999.

The mode couplings presented here agree with results found for large plasma crystals. The resonances and instabilities are very similar to those found by Liu et al. [1] and Couëdel et al. [2]. Also, the phenomenon where all modes would oscillate at the resonant frequency is very similar to the non-dispersive mode observed



Figure 9: Modes 15 and 23 are resonating, causing most other modes to vibrate at the resonant frequency.

by Liu et al. [1] and Couëdel et al. [2], where there were oscillations in all directions, and the frequency of the oscillations was independent of wave number. In addition, while frequency gaps due to mode couplings have not yet been found experimentally, the frequency gap caused by the coupling between the kinking and vertical relative modes agrees with a theoretical prediction by Yaroshenko et al. [3].

The results from the ten-particle systems clearly show that crystal structure has an impact on the stability of the crystals. While the two crystal structures had some modes in common, like the rotation and sloshing modes, most modes in one crystal structure were not in the other. This results in different couplings and therefore different confinements for which the systems are unstable.

5 Conclusion

Using molecular dynamics simulations of three- and ten-dust particle systems levitated in a plasma that included the ion wake effect, it was found that the ion wake effect leads to couplings between modes, and the mode couplings usually lead to resonances, and often to instabilities in the crystal. In addition, the coupling between the kinking and vertical relative mode lead to a frequency gap that prohibited resonance from occuring. Frequency gaps have not been observed in large crystals before.

Also, the ion wake effect leads to the appearance of a non-dispersive mode that was experimentally observed for large crystals by Liu et al. [1] and Couëdel et al. [2]. This mode sometimes appears when two coupled modes resonate together; the frequency of the non-dispersive mode is the resonant frequency. Overall, it seems that the ion wake effect affects small plasma crystals the same way that it affects large plasma crystals.

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