BAYLOR {mathematics}

**FALL 2010 NEWSLETTER**

# A Message from the Chair

Greetings, math alums, from the Department of Mathematics!

Since our last newsletter was released in January 2010, there have been significant changes at Baylor. Judge Kenneth Winston Starr has been inaugurated as Baylor University's 14th President and Dr. Elizabeth Davis was named Executive Vice President and Provost. All of us at Baylor are thrilled with these two appointments and, through the stability that these appointments bring, we are confident that Baylor will continue its push onwards and upwards to becoming one of the nation's elite universities.

On the departmental level, we have seen several important changes in the past few months. We've added Dr. Matthew Beauregard (University of Arizona), Gail Brooks (Baylor University, McLennan Community College), and Dr. Jonatan Lenells (University of Lund, Sweden) to our staff this fall and we are very pleased to welcome each of them into our mathematical family.

We are also excited to have Dr. Edward B. Burger, the 2010 Robert Foster Cherry Award winner for Great Teaching, with us this semester. Ed is the Lissack Professor for Social Responsibility and Personal Ethics at Williams College (MA) and is a multi-honored teacher of mathematics and an award-winning author of textbooks and videos. Besides teaching two courses for us, Ed is heavily involved with several other projects across our campus.

## HOMECOMING 2010

Homecoming this year is Saturday October 23. The Department of Mathematics will host a breakfast from 9:30-11:30 that morning on the first floor of Sid Rich. We would love to see you again and hear about your recent successes. Please stop by and visit with us.

In late September 2010, the National Research Council report, which analyzes the doctoral programs in our nation's universities, was released. Baylor has one of the youngest doctoral programs in mathematics in the country (having started in 2002). Given this fact – or in spite of this - it is remarkable how well our program has done in this short time. The NRC has us ranked sixth in the Big 12.

In August, the Meyerhoff Computer Classroom in mathematics was dedicated. Jim and Lisa Meyerhoff (both BS '78, MS '83) made a significant financial contribution to the Department to build this computer

classroom in Sid Richardson (SR 203). This classroom will allow the department to teach several courses in mathematics and mathematics education using various kinds of sophisticated software and new technology. We are very grateful to the Meyerhoffs for their vision and generous support.

As always, we have several distinguished lecturers visiting the department this semester. On October 18-19, William Dunham (Truman Koehler Professor of Mathematics, Muhlenberg College) will deliver two lectures in the third annual Baylor Undergraduate Series in Mathematics. Professor Dunham is a best-selling author and a leading mathematical historian in the country. His lectures are entitled *Newton and Leibniz: Mathematicians at War* and *Euler's Amicable Numbers*. For further information, see our webpage http://www.baylor.edu/math/index.php?id=57496.

Brian Pennington (BS, '84) will deliver the second annual lecture in the Life Experiences in Mathematics series on November 11. The title of Brian's talk is *Using Mathematics to Understand Financial Risk: Necessary but Not Sufficient*. Brian and his family live in Greenwich, CT; he is Director of Risk & Quantitative Resources for GoldenTree Asset Management in New York City and a fervent supporter of Baylor University.

Lastly, a big project that I am still working on is to complete a database of our math alumni and link it to our departmental web page. I would love to hear from each of our math alumni – pictures and bio sketches are most welcome!

# Department News

### Math Department Welcomes New Faculty

On September 1, the annual fall faculty banquet was held at the Clifton House in Waco. Among the guests were the department's new faculty members Matthew Beauregard, Gail Brooks, Edward Burger, and Jonatan Lenells.


**Matthew Beauregard**


**Gail Brooks**


**Ed Burger**


**Jonatan Lenells**

Matthew Beauregard is a new post doctoral visiting professor in the department. He joins us from the University of Arizona, where he earned his Ph.D. degree in 2008 under the supervision of Michael Tabor. Matt's current interests in mathematics include numerical analysis, scientific computing, mathematical modeling, traffic flow, and continuum mechanics. His hobbies include triathlon competitions, reading, and spending time with his wife, Becca, and son Caleb, and new baby daughter Adeline.

Gail Brooks is a new part-time lecturer in the department. She earned her M.S. in mathematics at Baylor University in 1994 under the direction of Frank Mathis. Gail has been a full-time faculty member in the Department of Mathematics at McLennan Community College since 2002. Prior to her appointment at MCC, she taught at Bosqueville High School for five years. Gail has led numerous professional

development classes, workshops, and presentations regarding incorporating streaming media into the classroom. She has also won several teaching awards, including a Master Teacher award in 2001 from Intel. Besides being the proud mother of son, Jimmy, and daughter, Becky. Gail is a dedicated and successful marathon runner. She has placed among the top 3 overall female finishers in five marathons since 2008, including 2 first place finishes.

Edward B. Burger is the recipient of the Robert Foster Cherry Award for Great Teaching and he will be in residence at Baylor for the fall semester. Ed is a Professor of Mathematics from Williams College where he is the Lissack Professor for Social Responsibility and Personal Ethics. Ed earned his Ph.D. degree in mathematics from the University of Texas at Austin in 1990. He is well recognized and highly respected throughout the world as both a first-rate research mathematician and a teacher extraordinaire. Dr. Burger has won a number of national awards for his research and teaching. Among these honors are several from the Mathematical Association of America including their National Distinguished Teaching Award (2000), the Polya Lectureship (2001-2003), the Chauvenet Prize (2004) and the Lester R. Ford Award (2006).

Jonatan Lenells is a new Assistant Professor in the department. His research focuses on nonlinear partial differential equations and their applications to mathematical physics. After completing his Ph.D. in Mathematics at Lund University in Sweden in 2006, he served as a Visiting Assistant Professor at the University of California, Santa Barbara (2006-2007) and as a Marie Curie Research Fellow at the University of Cambridge, UK (2007-2009). Before joining the Baylor faculty in 2010, he spent one year (2009-2010) at the Gottfried Wilhelm Leibniz University in Germany. Besides mathematics, he enjoys playing all kinds of sports: swimming, biking, table tennis, floorball, etc. He also likes playing piano and guitar, camping, juggling, and going to church. Interspersed with his doctoral studies, he spent a total of more than one year overseas with the organization Youth With A Mission, doing relief and missions work.

## Mathematics students win NSF Graduate Research Fellowship and Goldwater Scholarship

Two outstanding Baylor students, who were good friends growing up together in Lubbock, have recently won major national scholarships to continue their education.

**Meaghan McNeill**

**Rachel Wilkerson**

Meaghan McNeill, who graduated in May 2010 as a double mathematics/biology major, has won a Graduate Research Fellowship from the National Science Foundation. Meaghan started graduate work at Rice University this fall to pursue her Ph.D. degree in biomedical engineering. For further information on Meaghan's successes and future plans, please see the recent Baylor news release.

Rachel Wilkerson, a junior University Scholar concentrating in applied mathematics and physics, won a prestigious Barry M. Goldwater Scholarship. Rachel has just returned from Hungary where she participated in the highly competitive Budapest Semester in Mathematics. For more details on Rachel's achievements, please read the news item on the Baylor web page.

## A Big Thank You to Dr. Ron Stanke

At the end of the spring 2010 semester, Dr. Ron Stanke stepped down as the Graduate Director for the department after serving six years in this role. During this time, the Department the department oversaw a major growth in the department's graduate program as well as major changes in the administrative structure of the program. Under Dr. Stanke's leadership, 17 students graduated in the department with their Ph.D. degrees. Thanks, Ron, for a job very well done!

**Ron Stanke**

**Mark Sepanski**

The Director of Graduate Studies is the department's representative and advocate to the Baylor Graduate School and plays an important role in the life of the department. The Graduate Director's responsibilities include, but are not limited to, recruitment, graduate course offerings, graduate curriculum, budget, organization Ph.D. qualifying and preliminary exams, and coordination of the mentoring program for teachers.

Dr. Mark Sepanski has assumed responsibility for the department's graduate program and continues the standard of excellence maintained so admirably by his predecessor. Many thanks to both Ron and Mark!

## Recent Ph.D. Graduates

Three students graduated with Ph.D.'s in mathematics during the last academic year: Davut Tuncer, Drew Pruett, and Brian Williams. Dr. Tuncer is now on the faculty at the University of Tennessee at Chattanooga, Dr. Pruett is a Postdoctoral Research Fellow at the University of Mississippi Medical Center, and Dr. Williams is a Senior Data Analyst at Analytical Mechanics Associates.

**Davut Tuncer**

**Brian Williams**

**Drew Pruett**

The titles of their dissertations are as follows.

Davut Tuncer "The Left-Definite Spectral Analysis of the Legendre-type Differential Equation" (Advisor: Lance Littlejohn)

Drew Pruett "Diagrams and Reduced Decompositions for Cominuscule Flag Varieties and Affine Grassmannians" (Advisor: Markus Hunziker)

Brian Williams "Indecomposability in Inverse Limits" (Advisor: David Ryden)

Congratulations and best wishes to Davut, Drew, and Brian.

## Mathematics Scholarship Recipients

The Mathematics Department awarded $108,310 in scholarships to mathematics students for the current academic year. The following Baylor students received this scholarship support. We are very grateful to the families that endowed these scholarships and we wish each of the chosen students continued success in the coming year!

**The John C. Lattimore Mathematics Scholarship Fund**: Kelsey Carpenter, Melissa Principe

**The Earl, Maxine, Max, and Anita Bodine Mathematics Scholarship Fund**: Kathryn Ewing, Janie Hoorman, Michal Kokta, Heather Levihn, Chelsea Mitcham

**The K. L. and Vivian Carter Mathematics Scholarship Fund**: Whitney Banik, Mikayla Chien, Ryan Cowan, Lauren Miksch, Chrystal Rogers, Tammie Sawyer, Lisa Wiley

**The Jerry Johnson Mathematics Scholarship Fund**: Myles Baker, Caroline Northedge, Adam Telatovich, Caitlyn Thelen, Christina Tripp, Kim Woodsum

**The Howard L. Rolf Mathematics Scholarship Fund**: Tammie Sawyer

**The Hickey Mathematics Scholarship**: Matthew Hrna, Courtney Novak, Jessica Schwind

**The Piziak Mathematics Scholarship Fund**: Matthew Hrna

**The Roy Donald Perry Memorial Endowed Scholarship Fund**: Jona Goodman, Deborah Ryan, Tammie Sawyer

**The Professor Albert Boggess Mathematics Scholarship Fund**: Sergio Molina

**The Ruth and Gene Royer Mathematics Scholarship Fund**: Caroline Clark, Erwin Gostomski, Sergio Molina, Zach Reece, Brittney Turner

**The Schultz-Werba Mathematics Scholarship Fund:** Erwin Gostomski, Sergio Molina, Zach Reece, Brittney Turner

The department usually solicits applications each March for the following year's scholarships. Students may pick up an application form in the Mathematics office (SR 317). For more information concerning scholarships, students may contact Brian Raines at 710-4382.

### Congratulations to Gail Brooks

Gail Brooks, Lecturer in the Department of Mathematics, is an avid marathon runner. At the recent Quad Cities Marathon, she placed 5th overall among women in a race with over 4800 contestants. The marathon begins and ends in downtown Moline, IL, and passes through Bettendorf, IA, Davenport, IA, Rock Island, IL, and the mighty Mississippi's Arsenal Island. Thus the course runs through 4 cities, 3 bridges, 2 states, and 1 island, all along the Mississippi River. This year's race took place on September 26th. Congratulations Gail!

**Gail Brooks**

### Mathematics Alumnus receives Clarendon Prize from Oxford University

Will Brian (B.S. '08) begins doctoral work in mathematics at Oxford University this fall. He is one of a few distinguished mathematics students to receive the Clarendon Prize, which, among other things, covers all fees and provides a stipend for living expenses for the duration of his doctoral studies. Mr. Brian graduated from Baylor as a university scholar with concentration in mathematics in May of 2008 and recently earned a M.S. degree in mathematics from Tulane University. Congratulations Will, and keep up the good work!

**Will Brian**

## Meyerhoff Computer Classroom Dedicated

James and Lisa Meyerhoff were the guests of honor on Tuesday, August 17, 2010 when the computer classroom that they funded was officially dedicated and opened in the Sid Richardson building. The classroom, located in SR 203, will now be known as the *"Meyerhoff Computer Classroom"*.

Jim and Lisa Meyerhoff both graduated from Baylor with their Bachelor's degrees in 1978; Jim earned his degree in mathematics while Lisa obtained her degree in geology. In 1983, they both received their Master's degrees in geology from Baylor. In 1990, Lisa received her juris doctorate from South Texas College of Law.

Both Jim and Lisa have built distinctive and successful careers. After working as a geophysicist for Amoco, Maxus Exploration, and Noble Energy, Jim is now partner and Exploration Manager for Krescent Energy in Houston. Lisa is a partner with Baker & McKenzie LLP in Houston.

**Jim and Lisa Meyerhoff**

**The Meyerhoff Computer Classroom**

After a few opening words by Lance Littlejohn, Chair of the Department of Mathematics, Dr. Lee Nordt, Dean of the College of Arts and Sciences, thanked the Meyerhoffs for their dedication to Baylor and for their generous donation to the mathematics department.

**Dean Nordt thanking the Meyerhoffs**

**Jim and Lisa officially opening the computer classroom**

The Meyerhoff computer classroom has 30 student computers and an instructor computer station. The computer desks, made by Computer Comforts in Kemah, Texas, is unique in the sense that the computer monitors can be stored beneath the surface of the desk. Each of the computers is formatted with the latest software that will be used in mathematics classes to teach calculus, differential equations, linear algebra, numerical analysis, and mathematics education.

"The Meyerhoff Computer Classroom is an important and necessary step in the department's continuing growth. The computers are equipped with the latest mathematical software that will help students better understand geometric shapes and aid in their understanding of various mathematical concepts. This classroom will also allow us to offer more graduate courses in numerical analysis and computational mathematics for our own graduate students as well as graduate students from other programs who take our classes", says Lance Littlejohn.

**Dean Nordt, Jim and Lisa Meyerhoff, Lance Littlejohn**

**Demonstrating the new technology with Shelley Brooks**

"Education in America is what it is today because so many people have given back to institutions of higher learning such as Baylor," said Jim Meyerhoff. "Because God has blessed Baylor alumni with great educations and the resources to give back to Baylor, it is important that they continue the tradition of helping current and future Baylor students obtain quality educations in a Christian environment."

**Baxter Johns, Cherry Professor Edward Burger, Ed Oxford, and Gene Tidmore**


**New signage outside of SR 203**

For further information, please see the recent Baylor news article .

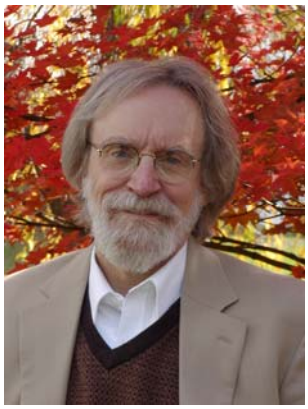
# Department Welcomes Distinguished Visitors to Speak in Lecture Series

### William Dunham headlines 2010 Undergraduate Lecture Series

Professor William Dunham, Truman Koehler Professor of Mathematics at Muhlenberg College in Allentown, Pennsylvania, will be the lecturer in the third annual Baylor Undergraduate Lecture Series in Mathematics. These lectures will take place October 18-19, 2010.

William Dunham received his B.S. (1969) from the University of Pittsburgh and his M.S. (1970) and Ph.D. (1974) from Ohio State. He has directed NEH-funded seminars on math history at Ohio State and has spoken on historical topics at the Smithsonian Institution, on NPR's "Talk of the Nation: Science Friday," and on the BBC. In 2008, as a Visiting Professor at Harvard University, he taught a course on the mathematics of Leonhard Euler.

Professor Dunham is both a popular lecturer and prolific author on the subject of the history of mathematics. He has written five best-selling books on this subject. In the 1990s, Dunham wrote three books - *Journey Through Genius* (1990), *The Mathematical Universe* (1994), and *Euler: The Master of Us All* (1999) - and in the present century he has done two more: *The Calculus Gallery: Masterpieces from Newton to Lebesgue* (2005) and *The Genius of Euler: Reflections on His Life and Work* (2007). Dunham's expository writing has been recognized by the MAA with the George Pólya Award in 1993, the Trevor Evans Award in 1997 and 2008, the Lester R. Ford Award in 2006, and the Beckenbach Prize in 2008. The Association of American Publishers designated *The Mathematical Universe* as the Best Mathematics Book of 1994.



**William Dunham**

The titles of Professor Dunham's lectures are *Newton and Leibniz: Mathematicians at War*, his public lecture, and *Euler's Amicable Numbers*, his colloquium lecture aimed at both mathematics majors and minors. For times and abstracts of his talks and further information on Professor Dunham, please click on the link Baylor Undergraduate Lecture Series in Mathematics.

## Brian Pennington to speak in 2010 Life Experiences in Mathematics Series

Brian Pennington (Baylor '84, Mathematics) will be the second speaker in the Life Experiences in Mathematics lecture series. His lecture, entitled *"Using Mathematics to Understand Financial Risk: Necessary but Not Sufficient"* will be given on November 11, 2010 from 4:00 – 5:00 pm in the Kayser Auditorim (Hankamer School of Business).

**Brian Pennington**

Brian received his Bachelor of Science degree in Mathematics from Baylor in 1984. While at Baylor, Brian was an honors program graduate and a member of Phi Beta Kappa, Mortar Board, and Sigma Chi fraternity. After graduation from Baylor, he studied finance and statistics at the University of Chicago

where he earned his MBA degree in 1986. Brian completed all of his course work and examinations for a Ph.D. in Finance and Statistics at Chicago.

Brian is a member of the Bear Foundation, Endowed Scholarship Society, and a life member of the Founders Associates and the Alumni Association. In 2001, Baylor honored Mr. Pennington and his wife, Amy, with the James Huckins Medallion in recognition of their support of the university.

Brian is currently Director of Risk & Quantitative Resources for GoldenTree Asset Management in New York City. Prior to joining GoldenTree, Brian Pennington was co-founder and Managing Partner of Rock Ridge Advisors L.L.C., a thematic global macro fund, where he was responsible for all aspects of management including capital commitment, quantitative resource development, risk measurement and business operations. Mr. Pennington began his investment career in 1986 at Salomon Brothers in the firm's renowned fixed-income proprietary trading unit and also headed the firm's yield curve and O.T.C options arbitrage. Mr. Pennington co-founded Convergence Asset Management, a registered commodity pool operator in 1997. In 2001, he joined Caxton Associates as a leading investment advisor, where he was also a member of the Risk management Committee for Caxton Relative Value Holdings.

Brian and his wife reside in Greenwich, Connecticut with their two daughters and son. To see a poster advertising Brian's lecture, please click here.

## George Andrews, President of the American Mathematical Society, speaks in 2010 Baylor Lecture Series in Mathematics

Professor George E. Andrews, Evan Pugh Professor of Mathematics at the Pennsylvania State University, spoke in the fourth annual Baylor Lecture Series in Mathematics on Wednesday, April 14, 2010.

The title of Professor Andrews' public lecture was entitled "*Ramanujan and His Amazing Lost Notebook*." His second lecture "*The World of q-Orthogonal Polynomials and Bailey Chains"* was delivered on Thursday, April 15. For further information on Professor Andrews and his visit, please click on our Baylor Lecture Series in Mathematics link.

**George Andrews**

Professor Andrews is the current President of the American Mathematical Society. He is also a member of the National Academy of Sciences and the American Academy of Arts and Sciences. He has been a Fulbright Scholar as well as a Guggenheim Fellow.

While visiting Trinity College in Cambridge (U.K.) in 1975, Professor Andrews discovered what is now known as 'Ramanujan's Lost Notebook.' This collection contains about 600 identities and equations that the famed and enigmatic Indian mathematician Srinivasa Ramanujan (1887-1920) had written during the last year of this life. Remarkably, yet mysteriously, nearly every formula in this book was written without

proof. Working for the past three decades, Andrews and his co-workers have been able to produce proofs for most of the formulae in the book. Andrews' work in this area was featured in a 1987 episode of the PBS show NOVA that was devoted to describing the life and work of Ramanujan.

Professor Andrews is also nationally recognized for his compelling essays, provocative lectures, and tireless work in mathematics education. He is deeply concerned about recent national trends in mathematics education, the use of technology in the classroom, and how mathematics is being taught at all levels.

## Baylor Hosts Panel Discussion on "Teaching Mathematics as a Career"

Cathy Seeley, past President of the National Council of Teachers of Mathematics (and a former plenary speaker at a recent Central Texas Council of Teachers of Mathematics conference in Waco) says that "being a teacher is the most important job on the planet, and mathematics opens doors to all kinds of options for students".

On September 16, Baylor University hosted a panel discussion on the topic of "Teaching Mathematics as a Career" to a standing-room only crowd in Draper 139. This was a joint venture between the Department of Curriculum and Instruction in the School of Education and the Department of Mathematics in the College of Arts and Sciences.

In the United States – and especially in Texas – there is a critical shortage of qualified elementary, middle, and high school teachers of mathematics. President Obama recently said "Teacher quality is the most important single factor that influences whether students succeed or fail in the STEM fields of science, technology, engineering, and math". And former President George W. Bush said that "global competitiveness starts with improved math and science education for American students".

Jointly promoted by the School of Education and the Department of Mathematics, this event was organized and coordinated by Dr. Baxter Johns (Department of Mathematics) and Dr. Douglas Rogers (Associate Dean, School of Education). The seven members of the panel were moderator Dr. Edward Burger (Visiting Professor in the Department of Mathematics and the Robert Foster Cherry Professor for Great Teaching), Dr. Tommy Bryan (Department of Mathematics), Dr. Rachelle D. Meyer (Department of Curriculum and Instruction), Oscar Perales (Graduate student), Karen Hassell (Waco ISD), Vicky Brown (Teacher, University High), and Christi Carson (Teacher, Lake Air Intermediate). Among some of the topics that the panel discussed were

• current and future opportunities in the teaching of mathematics

• degree programs for prospective mathematics teachers

• certification issues

Since the panel consisted of teachers ranging from elementary school to the university level, part of the discussion between the panel and the audience centered around what it is like to be a teacher in mathematics from K-20.

A special thanks to Carolyn Muska, Associate Director of Career Services for her assistance and for providing refreshments and pizza! We hope that this event will become an annual one!

# The Method of Equal Proportions: Apportioning Seats of the United States House of Representatives

*Kathy Hutchison, Ed Oxford*

The method currently used to apportion seats in the United States House of Representatives might one day end in a tie when it is applied to census data. Prior planning regarding such an event could help avoid an unpleasant political confrontation. In this article, a description of why a tie can happen is presented, and three simple suggestions for breaking a tie are offered.

The method currently used is called the Method of Equal Proportions. Following the census of 1940, Congress adopted this method to apportion seats in the House of Representatives. In 1928 E. V. Huntington described a procedure for implementing the method [2], and a version of this procedure is still used by the Census Bureau [3].

E. V. Huntington

Let $k$ denote the number of states and assume there is an ordered arrangement of the states where the $i$-th state has population $P_i$ and $P_1 \geq \ldots \geq P_k$. If $H$ is the size of the House of Representatives, it is necessary to require that $H \geq k$ in order to satisfy the constitutional mandate that each state must be apportioned at least one seat. Define $d(a) = \sqrt{a(a+1)}$ for each nonnegative integer $a$. Then $a \leq d(a) < a+1$ for each $a$. The array

| | | | |
|---|---|---|---|
| $\frac{P_1}{d(1)}$ | $\frac{P_1}{d(2)}$ | $\frac{P_1}{d(3)}$ | $\cdots$ |
| $\frac{P_2}{d(1)}$ | $\frac{P_2}{d(2)}$ | $\frac{P_2}{d(3)}$ | $\cdots$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $\frac{P_k}{d(1)}$ | $\frac{P_k}{d(2)}$ | $\frac{P_k}{d(3)}$ | $\cdots$ |

is called the Huntington array associated with the population vector $(P_1,\cdots,P_k)$. The values in the cells of this array are the priority values associated with the population vector $(P_1,\cdots,P_k)$. Since $d$ is a strictly increasing function, each row of the Huntington array is strictly decreasing. Moreover, each column of the Huntington array is monotonically decreasing because $P_1 \geq \ldots \geq P_k$. The values in the cells of the Huntington array can be arranged as a monotonically decreasing sequence $V_1, V_2, V_3, \ldots$ where a value is

listed as many times as it appears in the cells of the array. The sequence $V_1, V_2, V_3, \ldots$ is called the priority-value sequence associated with the population vector $(P_1, \cdots, P_k)$.

On web site [3] is a table of four columns with headers: house seat, state, state seat, and priority value. The priority-value column contains the priority-value sequence defined above. The column headed by "house seat" contains the integers 51, 52, $\cdots$, 440. The state with the largest priority value gets the 51st house seat, the state with the second largest priority value gets the 52nd house seat, etc... In this way, the Census Bureau associates a state with each house seat numbered 51 through 440. The Bureau goes a little beyond the current house size of 435 because Congress might decide to enlarge the size of the House of Representatives. This association of a state with each house seat is uniquely defined only in the case that the finite priority-value sequence $V_1, V_2, V_3, \ldots, V_{H-k}, V_{H-k+1}$ is strictly decreasing.

To illustrate Huntington's priority-value working rule in the case where $V_1, V_2, V_3, \ldots, V_{H-k}, V_{H-k+1}$ is strictly decreasing, consider a country with five states A, B, C, D, and E and with populations 5000, 4500, 3200, 2000, and 1500, respectively. Initially each state is allotted one seat, so the apportionment vector corresponding to house size 5 is (1,1,1,1,1). The first four terms, $V_1, \ldots V_4$ of the priority-value sequence are used to find the apportionment vector corresponding to each house size $6, \cdots, 9$. The terms $V_1, \ldots V_5$ are $\frac{5000}{\sqrt{2}}, \frac{4500}{\sqrt{2}}, \frac{3200}{\sqrt{2}}, \frac{5000}{\sqrt{6}}, \frac{4500}{\sqrt{6}}$ respectively. Since state A corresponds to the first term of the priority-value sequence, state A is allotted the sixth seat and the apportionment vector corresponding to house size six is (2,1,1,1,1). State B corresponds to the second term of the priority-value sequence and is allotted the seventh seat. The apportionment vector corresponding to house size seven is (2,2,1,1,1). At this stage, $\frac{3200}{\sqrt{2}}$ is the largest unused priority value and it corresponds to state C. Consequently, state C is allotted the eighth seat and (2,2,2,1,1) is the apportionment vector that corresponds to house size eight. Since $\frac{5000}{\sqrt{6}}$ is the next largest priority value, state A is allotted the ninth seat. So (3,2,2,1,1) is the apportionment vector that corresponds to house size nine.

There is a shortcut whenever $V_{H-k} \neq V_{H-k+1}$. If $C$ is a positive real number, then $\frac{P_1}{C}, \cdots, \frac{P_k}{C}$ are called the quotas associated with $C$. If $a$ is a nonnegative integer and $a \leq \frac{P_i}{C} \leq a+1$ then $\frac{P_i}{C}$ rounds to $a+1$ when $\frac{P_i}{C} > d(a)$ and rounds to $a$ when $\frac{P_i}{C} < d(a)$. The goal is to find a common divisor $C$ so that the sum of the values to which $\frac{P_1}{C}, \cdots, \frac{P_k}{C}$ round is the house size $H$. Fortunately, any value strictly between $V_{H-k}$ and $V_{H-k+1}$ gives a common divisor where the sum of the rounded values is $H$. Moreover if $\frac{P_i}{C}$ rounds to $a_i$, then $(a_1, \cdots, a_k)$ is the unique apportionment vector associated with house size $H$. This procedure of finding a common divisor and rounding is called the traditional rule of apportionment. The traditional rule can be used with $C = 820$ to show (6,6,4,2,2) is the apportionment vector corresponding to the population vector (5000,4500,3200,2000,1500) with house size 20 because $V_{15} = \frac{4500}{\sqrt{30}} > 820 > \frac{2000}{\sqrt{6}} = V_{16}$.

The priority-value working rule associates apportionment vectors with each house size greater than or equal $k$. The house size $k+j$ corresponds to the term $V_j$ of the priority-value sequence. The apportionment vectors associated with house size $k+j$ depend on properties of the plateau (a

subsequence with constant value where no longer constant subsequence has that value) of the priority-value sequence in which $V_j$ sits and on the unique apportionment vector that corresponds to the house size just prior to the plateau that contains $V_j$.

Assume $V_j$ is the first term of a plateau of length $s$ of the priority-value sequence and that there is a unique apportionment vector associated with house size $k+j-1$. If $s \geq l$, one gets the $\binom{s}{l}$ apportionment vectors for house size $(k+j-1)+l$ by adding one to exactly $l$ of the coordinates, corresponding to the requisite $s$ states, of the unique apportionment vector for house size $k+j-1$. Observe that this rule gives $\binom{s}{s}$ apportionment vectors for house size $(k+j-1)+s$. Consequently if there is a unique apportionment vector corresponding to the house size just prior to the beginning of a plateau of the priority-value sequence, there is a unique apportionment vector corresponding to the house size at the end of that plateau.

To illustrate the priority-value working rule when $V_{H-k} = V_{H-k+1}$ apportion 60 seats to three states with populations 3500000, 600000, and 100000, respectively. Some terms of the priority-value sequence are $V_{55} = \frac{3500000}{\sqrt{48*49}} \approx 72168.78$ and $V_{56} = V_{57} = V_{58} = \frac{3500000}{\sqrt{49*50}} = \frac{600000}{\sqrt{8*9}} = \frac{100000}{\sqrt{1*2}} \approx 70710.68$. Since $V_{58-3} > V_{58-3+1}$, the traditional rule can be used to show (49,8,1) is the apportionment vector for house size 58. Since $V_{56} = V_{57} = V_{58}$, the three states are equally entitled to the 59th house seat. So the apportionment vectors for house size 59 are (50,8,1), (49,9,1), and (49,8,2). The three apportionment vectors for house size 60 are (50,9,1), (50,8,2), and (49,9,2).

It may be disconcerting to obtain multiple apportionment vectors for certain house sizes. However, this outcome may be more palatable when associated with Huntington's pairwise comparisons of states' representations. See [2] and [1, pp. 100-104] for details about Huntington's pairwise comparisons. Let $a_i$ denote the number of seats tentatively apportioned to state $i$ and let $P_i$ denote the population of state $i$. As presented in [1, p. 102] and when using the Method of Equal Proportions, the amount of inequality between states $i$ and $j$ is $\frac{a_i P_j}{a_j P_i} - 1$ whenever $\frac{a_i}{P_i} \geq \frac{a_j}{P_j}$. If state $i$ is over-represented in comparison to state $j$, that is if $\frac{a_i}{P_i} \geq \frac{a_j}{P_j}$, then a transfer of a seat from state $i$ to state $j$ should be made if the amount of inequality is diminished. Huntington proves [2] that by making successive transfers that diminish the amount of inequality, " it is always possible to arrive at a final apportionment which cannot be 'improved' by any further transfer between two states." The apportionment vectors given by the priority-value working rule and associated with house size 60 and the population vector (3500000,600000,100000) are: (50,9,1), (50,8,2), and (49,9,2). These are precisely the apportionment vectors for house size 60 which do not admit a transfer between two states that diminishes the amount of inequality.

In case of the improbable event that a tie were to occur, in other words were $V_{H-k} = V_{H-k+1}$, what would be a good way to break the tie? Three suggestions are offered:

(1) If $V_{H-k} = V_{H-k+1}$, then temporarily reduce the house size to the house size just prior to the plateau that contains $V_{H-k}$. There is a unique apportionment vector corresponding to this adjusted house size.

(2) If $V_{H-k} = V_{H-k+1}$, then $V_{H-k}$ belongs to a plateau of the priority-value sequence that has length $s \geq 2$. When $V_{H-k}$ is the $l$ -th term of this plateau, randomly select $l$ of the requisite $s$ coordinates of the unique apportionment vector that corresponds to the house size just prior to the plateau that contains $V_{H-k}$ and add exactly one to each of these $l$ coordinates. This yields an apportionment vector for house size $H$ .

(3) If $V_{H-k} = V_{H-k+1}$ then temporarily increase the house size to the house size corresponding to the end of the plateau that contains $V_{H-k}$. There is a unique apportionment vector corresponding to this adjusted house size.

Cursory observations indicate that the likelihood of a tie (namely of $V_{H-k} = V_{H-k+1}$ ) when applying the Method of Equal Proportions to the decennial census is very small. However, no credible measure of this likelihood has been found.

[1] M.L. Balinski and H.P. Young, *Fair Representation*, 2nd edition, Brookings Institution, Washington D.C., 2001

[2] E.V. Huntington, The Apportionment of Representatives in Congress, *Transactions of the American Mathematical Society*, 30, no. 1:85-110, 1928

[3] www.census.gov/population/censusdata/apportionment/00pvalues.txt

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