Mutual Fund Style Analysis: A Stochastic Dominance Approach

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Abstract

It is a well-known fact that actively managed mutual funds on average underperform passive benchmarks. In this paper, we use the stochastic dominance test proposed by Linton, Maasoumi, and Whang (2005) to shed new light on mutual fund performance on average and across styles. This test evaluates mutual fund performance using a non-parametric framework that 1) imposes a minimal set of conditions on preferences; and 2) analyzes the entire return distribution for each mutual fund group. We find little evidence that actively managed mutual funds on average underperform the passive benchmark, suggesting that mutual fund performance results are highly sensitive to investor preference assumptions. Exploring the returns for different styles of mutual funds, we find that aggressive mutual funds underperform the market for risk-averse investors, whereas both growth & income and income funds outperform the market for prudent investors. Furthermore, we find that mutual fund portfolios formed by the stochastic dominance approach provide superior future performance.

Key Words: Mutual Fund, Stochastic Dominance, Performance Evaluation

JEL Classification: C12,C15,G11

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1 Introduction

Mutual funds are one of the fastest growing financial intermediaries in the United States. The industry has grown in size to 16 trillion dollars and attracts over 40 percent of U.S. households as investors. It is the second largest type of financial intermediary in the United States, falling just short of commercial banks. However, there has been a debate about whether or not actively managed mutual fund managers add value. The answer to this questions is crucial for investors’ asset allocation decisions and asset managers’ investment strategies. Academics find that the growth in actively managed U.S. equity mutual funds is puzzling since numerous studies have shown that, post fees, these funds provide investors with average returns significantly below those on passive benchmarks. While most previous research concludes that actively managed mutual funds underperform the market when comparing the mean and standard deviation of returns, this paper asks two questions: 1) Can some omitted risk factors or investors’ preferences explain the puzzle? 2) Do some styles of actively managed mutual funds perform better than others or better than the market?

Investors and academic researchers have a long-standing interest in return and risk tradeoff. The Sharpe ratio, which is defined as the ratio of excess return to volatility, is one of the most common measures of portfolio performance. Sharpe (1966) developed it as a tool for mutual fund performance evaluation. However, Goetzmann, Ingersoll, and Spiegel (2007) point out that a dynamic levering strategy, which involves increasing leverage after a period of poor returns or decreasing leverage after a period of good returns, could increase the Sharpe ratio. The manipulation of the Sharpe ratio consists largely in selling the upside return potential, thus creating a distribution with high left-tail risk. A significant restriction on the applicability of the Sharpe ratio results from the facts that: 1) It assumes a quadratic utility function; and 2) It utilizes only the first two moments of the return distributions.

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When the underlying data appear to follow a normal distribution, quadratic preferences will not miss anything by only considering mean and variance. However, it is well-known that the distributions of financial returns deviate significantly from normality. Thus, variance is inadequate as the only quantifier of risk in mutual fund performance evaluation.

High distribution moments have received notable attention after the recent financial turmoil. A growing body of research reveals that investors favor right skewness, and do not like tail risk or rare disaster risk. Sortino and Price (1994), Dowd (2000), and Kadan and Liu (2014) propose performance measures that account for the higher moments of the distribution. In this paper, we study a performance measure that not only accounts for higher moments of the distribution but also imposes a minimal set of conditions on investors’ preferences.

This paper uses a stochastic dominance (SD) approach to test if mutual funds on average underperform as a group and if particular styles of mutual funds underperform. The main advantages of the stochastic dominance approach are that it imposes a minimal set of conditions on investors’ preferences and the underlying return distributions. These conditions consist of degree of risk aversion, preference for skewness, and an aversion to kurtosis. For a rational agent with a known utility function, one group of mutual funds is preferred if it maximizes expected utility, which works in theory. However, in practice it is often difficult to find an investor’s utility function. Therefore, it would be most useful to know whether or not a certain group of mutual funds is the dominant choice because it is preferred by all agents whose utility functions share certain general characteristics.

To implement the stochastic dominance approach, we examine various levels of stochastic dominance between the returns on mutual fund groups and the passive benchmark. The rules

\[^3\] For example, Mandelbrot (1963) and Breen and Savage (1968) have shown that stock price changes are inconsistent with the assumption of normal probability distributions.

\[^4\] See for example, Kraus and Litzenberger (1976), Jean (1971), Kane (1982), Harvey and Siddique (2000), Zhang (2005), Smith (2007), Brunnermeier, Gollier, and Parker (2007), Boyer, Mitton, and Vorkink (2010), Kumar (2009), and others.

\[^5\] See for example, Barro (2009), Gabaix (2008), Gourio (2012), Chen, Joslin, and Tran (2012), Wachter (2013), and others.
for first order stochastic dominance (FSD) state the necessary and sufficient conditions under which one asset is preferred to another by all expected utility maximizers. The rules for second order stochastic dominance (SSD) state the necessary and sufficient conditions under which one asset is preferred to another by all risk-averse expected utility maximizers. The rules for third order stochastic dominance (TSD) state the necessary and sufficient conditions under which one asset is preferred to another by all prudent (increasing risk aversion) risk-averse expected utility maximizers. If there is no dominance relationship between different classes of mutual funds and the passive benchmark, it suggests that investors with different utility functions will have different preferences over mutual funds and the passive benchmark. If the passive benchmark was to dominate certain mutual fund groups at the first order (or second order), it would mean that all expected utility maximizers (risk-averse investors) prefer the passive benchmark to certain classes of mutual funds. This outcome would be quite puzzling. Why would investors continue to pour money into actively managed funds despite the fact that they prefer the distribution of the passive benchmark?

Using a stochastic dominance approach, which imposes a minimal set of conditions on investors' preferences and the underlying return distributions, we find little evidence that actively managed mutual funds on average underperform the passive benchmark. Although aggressive mutual funds underperform the market for risk-averse investors, there is some evidence showing that both growth & income as well as income funds outperform the market for prudent investors. These results indicate the importance of considering investors' utility functions when analyzing investor behavior.

To implement the stochastic dominance approach, we first compare the return distributions between the mutual funds and the passive benchmark. We adopt value-weighted returns of all stocks listed on the NYSE, AMEX, or NASDAQ (market) as the passive benchmark for comparison. Over the period of 1980 to 2015, there is no evidence of a first order stochastic dominance relationship between the mutual funds and the market. This indicates that expected utility maximizers do not all prefer either mutual funds or the passive benchmark.
Similarly, there is no evidence of a second order or third order stochastic dominance relationship between the mutual funds and the market. These results show that there is no uniform preference between the mutual funds and the market for all risk-averse investors nor for all prudent investors as well.

Second, we examine whether some styles of mutual funds perform better than others or than the market. Mutual funds attempt to differentiate their services by specializing in certain sectors of the stock market. Chen, Jegadeesh, and Wermers (2000) point out that growth funds claim to specialize in the “glamour” or low book-to-market stocks, while income funds claim to specialize in “value” or high book-to-market stocks. We analyze whether such specialization adds value to investors and whether some styles of actively managed mutual funds perform better than others or better than the market. We analyze the return distribution of four classes of mutual fund investment objectives (aggressive, growth, growth & income, and income). After deducting management fees, we find that the market dominates the aggressive fund by second order stochastic dominance from 1980 to 2015. This suggests that all risk-averse investors prefer the market over average aggressive funds. The result confirms that it is indeed puzzling why risk-averse individuals would invest in aggressive funds. However, it is possible that the major flow to aggressive funds is made by investors with certain non-concave utility functions.

Surprisingly, there is some evidence showing that both income and growth & income funds dominate the market by third order dominance before and also after fees are deducted. In addition, the SD results show that income and growth & income funds dominate the market by second order dominance during economic recessions. This result is consistent with the findings in Moskowitz (2000), Kosowski (2006) and Glode (2011): active mutual funds perform better in recessions and are therefore potentially desirable relative to passive benchmarks.

Third, we calculate the risk adjusted return based on a four-factor model in order to further compare the performance among different classes of mutual funds. Using a four-
factor model, a number of previous studies document that the typical actively managed U.S. equity fund earns a negative alpha after fees (Gruber (1996), Carhart (1997), French (2008), and Fama and French (2010)). We confirm this finding in our risk adjusted return estimation as well. After controlling for the market risk premium, size, value, and momentum factors, the risk adjusted return of aggressive funds is dominated by all of the other three classes of mutual funds by second order stochastic dominance. In addition, growth & income funds dominate all of the other three classes of mutual funds by second order stochastic dominance.

Overall, our results indicate that SD tests provide a robust analysis of mutual fund performance. From a broader perspective, there are two important issues for investors to consider when selecting mutual funds: whether a superior mutual fund can be identified in advance and whether there is persistence in performance. A number of empirical studies demonstrate that the relative performance of equity mutual funds persists from period to period.\(^6\)

Finally, we examine whether ex-post SD relationships provide exploitable information on ex-ante returns. We construct mutual fund portfolios based on second order stochastic dominance. At the beginning of each year between 1995-2015, we identify the dominated (second order) mutual funds based on the most recent sixty monthly returns. We then form an equal weighted portfolio of these dominated mutual funds, which is rebalanced annually. The results show that portfolios formed by a stochastic dominance approach deliver better performance than mean-variance efficient portfolios.

Although a number of studies have used a stochastic dominance approach to rank return distributions in the finance literature, most of these SD tests do not take the dependence structure of financial returns into account. Lean, Phoon, and Wong (2011) employ a stochastic dominance approach to rank the performance of commodity trading advisers’ funds. Seyhun (1993) uses a stochastic dominance approach to test for the existence of the January effect. The critical value of stochastic dominance tests in these two studies require

an i.i.d assumption for returns. However, Fung and Hsieh (1997) and Brown and Goetzmann (1995) show mutual fund returns are highly correlated and this cross-fund correlation issue should be addressed. In this paper, we have adopted the Linton, Maasoumi, and Whang (LMW) test, which can accommodate not only the general dependence between mutual fund returns, but also the serial dependence.

We describe our data in detail in Section II. Section III introduces the stochastic dominance test, and Section IV discusses the hypotheses and test statistics. Empirical results are provided in Section V. Section VI discusses robustness tests and Section VII concludes.

2 Data

Our sample builds upon two data sets. We begin with a mutual fund sample from the CRSP (Center for Research in Security Prices) Survivorship-Bias-Free Mutual Funds database. The database includes information on funds' returns, fees, investment objectives (style), and size (total net assets). In this study, we limit our analysis to actively managed domestic equity mutual funds between March 1980 and December 2015, which contains the most complete and reliable return data.7 Specifically, we include only mutual funds that have a self-declared investment objective of “MCG,” “AGG,” “CA,” “G,” “LTG,” “GRO,” “IEQ,” “OPI,” “EI,” “GCI,” “GRI,” or “GI.”

We follow Kacperczyk, Sialm, and Zheng (2008) in eliminating balanced, bond, money market, international, sector, and index funds. We mainly use CRSP objective codes to classify the mutual funds into four investment classes (aggressive, growth, growth & income, and income). As shown in Table 1, we classify mutual funds with the objective of “Max-

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7Fama and French (2010) state that there is a potential problem in the CRSP mutual fund return data during the period 1962 to 1983. For this time period, about 15% of the funds on the CRSP report only annual returns, and the average annual equal-weight (EW) return for these funds is 5.29% lower than for funds that report monthly returns. Also, MFLINKS data starts in March 1980. Given the nature of our tests and data availability, we choose the sample period from March 1980 to December 2015.
imum Capital Gains,” “Equity USA Aggressive Growth,” or “Capital Appreciation Funds” as aggressive funds; mutual funds with the objective of “Growth,” “Long-Term Growth,” or “Equity USA Growth” as growth funds; mutual funds with the objective of “Equity Income,” “Option Income,” or “Equity Income Funds” as income funds; and mutual funds with the objective of “Growth and Current Income,” “Equity USA growth & income,” “Equity USA Income & Growth,” or “Growth and Income Funds” as growth & income funds.

Some mutual funds have multiple share classes. The CRSP data lists each share class as a separate fund. Different share classes have the same holding compositions and typically differ only in fee structure. The returns histories are therefore sometimes duplicated in the CRSP dataset. For example, if a fund started in 1983 and split into four share classes in 1993, each new share class of the fund is permitted to inherit the entire return history. This can create a bias when averaging returns across mutual funds. For funds with multiple share classes, we use the identification code in MFLINKS to combine different classes of the same fund into a single value-weighted fund. Wermers (2000) provides a description of how MFLINKS are created. Each monthly fund return is computed by weighting the return of its component share classes by their beginning-of-month total net asset values.

We obtain monthly data for the size, value, momentum, and market portfolios for the period of 1980 to 2015 from Kenneth French’s data library. We measure recessions using the definition of the National Bureau of Economic Research (NBER) business cycle dating committee. The start of the recession is the peak of economic activity and its end is the trough. Our aggregate sample spans 430 months of data from March 1980 until December 2015, among which 55 are NBER recession months (13%).
3 Stochastic Dominance

This section provides a non-parametric approach based on stochastic dominance testing to evaluate mutual fund performance. The theory of stochastic dominance offers a decision-making rule under uncertainty provided the decision maker’s utility function has certain properties. The different orders of stochastic dominance correspond to increasing restrictions on the shape of the utility function and the agents’ attitude towards higher order moments. These restrictions are non-parametric and do not require specific parametric function forms.

We first briefly define the criteria of stochastic dominance:

1. First order stochastic dominance: When A dominates B by first order stochastic dominance, all expected utility maximizers \( u' \geq 0 \) prefer A to B.

2. Second order stochastic dominance: When A dominates B by second order stochastic dominance, all risk-averse expected utility maximizers \( u' \geq 0, u'' \leq 0 \) prefer A to B.

3. Third order stochastic dominance: When A dominates B by third order stochastic dominance, all prudent risk-averse expected utility maximizers \( u' \geq 0, u'' \leq 0, u''' \geq 0 \) prefer A to B.

We use \( X_1 \) and \( X_2 \) to denote two random variables (e.g., mutual fund returns and market returns). Let \( U_1 \) denote the set of von Neumann-Morgenstern type utility functions, \( u \), such that \( u' \geq 0 \) (more is better than less). Let \( U_2 \) denote the set of utility functions in \( U_1 \) for which \( u'' \leq 0 \) (concavity). Let \( U_3 \) denote the class of all utility functions in \( U_2 \) for which \( u''' \geq 0 \) (increasing risk aversion). Let \( F_1(x) \) and \( F_2(x) \) be the cumulative distribution functions, respectively.

Then define the following:
Definition 1: $X_1$ first order stochastic dominates $X_2$, denoted $X_1 \succeq_{FSD} X_2$, if and only if:

$E[u(X_1)] \geq E[u(X_2)]$ for all $u \in U_1$ with strict inequality for some $u$; or

$F_1(x) \leq F_2(x)$ for all $x$ with strict inequality for some $x$.

Definition 2: $X_1$ second order stochastic dominates $X_2$, denoted $X_1 \succeq_{SSD} X_2$, if and only if:

$E[u(X_1)] \geq E[u(X_2)]$ for all $u \in U_2$ with strict inequality for some $u$; or

$\int_{-\infty}^{x} F_1(t)dt \leq \int_{-\infty}^{x} F_2(t)dt$ for all $x$ with strict inequality for some $x$.

Definition 3: $X_1$ third order stochastic dominates $X_2$, denoted $X_1 \succeq_{TSD} X_2$, if and only if:

$E[u(X_1)] \geq E[u(X_2)]$ for all $u \in U_3$ with strict inequality for some $u$; or

$\int_{-\infty}^{x} \int_{-\infty}^{z} F_1(t)dtdz \leq \int_{-\infty}^{x} \int_{-\infty}^{z} F_2(t)dtdz$ for all $x$ with strict inequality for some $x$.

Mathematically, lower order dominance implies all higher order dominance rankings. In the case of first order dominance, the distribution function of $X_1$ lies everywhere to the right of the distribution function of $X_2$ except for a finite number of points where there is strict equality. For first order stochastic dominance, the probability that returns of $X_1$ are in excess of $r$ is higher than the corresponding probability associated with $X_2$.

$$Pr(X_1 > r) \geq Pr(X_2 > r).$$

An important feature of the definitions of stochastic dominance is that they impose minimum conditions on the preferences of agents within the class of von Neumann–Morgenstern utility functions. Stochastic dominance is more satisfactory than the commonly used mean-variance rule since it is defined with reference to a much larger class of utility functions.
and return distributions. Levy (2006) provides an example showing that the mean-variance approach produces an inaccurate evaluation result. Suppose that \( X_1 \in \{1, 2\} \) has equal probability on each outcome and that \( X_1 \in \{2, 4\} \) also has equal probability on each outcome. Then \( E(X_1) < E(X_2) \), but \( var(X_1) < var(X_2) \), so that there exists a mean-variance optimizer who prefers \( X_1 \) over \( X_2 \). However, this does not make economic sense because \( X_1 \leq X_2 \) with a probability of one. \( X_1 \) is first order stochastic dominated by \( X_2 \).

4 Hypotheses and Test Statistics

\( X_1 \) denotes the average actively managed mutual fund return; \( X_2 \) denotes the market return; \( X_3 \) denotes the aggressive fund return; \( X_4 \) denotes growth fund return; \( X_5 \) denotes growth and income fund return; and \( X_6 \) denotes income fund return. The hypothesis tested is whether or not one group of mutual funds or the market dominates the other. We examine the stochastic dominance relationship between all pairs of returns of \( X_k \) for \( k = 1 \ldots 6 \). One example of the type of test we conduct is:

\( H_0: \) The market stochastically dominates average actively managed mutual fund, with the alternative being that there is no stochastic dominance.

Next, we formalize these tests. Let \( \chi \) denote the support of \( X_k \) for \( k = 1 \ldots 6 \) and let \( s = 1, 2, 3 \) represent the order of stochastic dominance. Define:

\[
F_K(x) = P(X \leq x),
\]

\[
D^{(1)}_K(x) = F_K(x),
\]
\[ D^{(s)}_K(x) = \int_{-\infty}^{x} D^{(s-1)}_K(t) dt \text{ for } s \geq 2. \] (3)

We say that \( X_k \) stochastically dominates \( X_l \) at order \( s \), if \( D^{(s)}_k(x) \leq D^{(s)}_l(x) \) for all \( x \) with strict inequality for some \( x \).

For each \( k = 1 \ldots 6; s = 1, 2, 3, \) and \( x \in \chi \), let \( D^{(s)}_{kl} = D^{(s)}_k(x) - D^{(s)}_l(x) \). Define:

\[ d^*_s = \max_{k \neq l} \sup_{x \in \chi} \left[ D^{(s)}_{kl} \right]. \] (4)

As Klecan, McFadden, and McFadden (1991) suggests, the hypothesis of interest can be stated as:

\[ H_0 : d^*_s \leq 0 \text{ vs. } H_a : d^*_s > 0. \] (5)

The test statistics are based on the empirical analogues of \( d^*_s \). We define the test statistics as:

\[ D^{(s)}_N = \max_{k \neq l} \sup_{x \in \chi} \sqrt{N} \left[ \bar{D}^{(s)}_{kl}(x) \right], \] (6)

where

\[ \bar{D}^{(s)}_{k}(x) = \frac{1}{N(S-1)!} \sum_{i=1}^{N} (x - X_{ki})^{s-1} 1(X_{ki} \leq x) \text{ for } k = 1, \ldots, 6. \] (7)

We adopt a recentering function to account for the effect of the parameter estimation error as suggested in Donald and Hsu (2013). Simulation results in Donald and Hsu (2013) show that the recentering function increases the power of the test. For a given small negative number \( a_N \), define the recentering function as \( \mu = (\hat{F}_k(x) - \hat{F}_l(x)) * 1(\sqrt{N}(\hat{F}_k(x) - \hat{F}_l(x)) < a_N). \)

We next describe the main method for obtaining critical values: the subsampling approach. Klecan, McFadden, and McFadden (1991) point out that even when the data are
i.i.d in stochastic dominance testing, the standard bootstrap method does not work because one needs to impose the null hypothesis in that case. The mutual dependence of the fund returns as well as the time series dependence in the data make it challenging to obtain consistent critical values. As Linton, Maasoumi, and Whang (2005) suggest, we use the subsampling method to obtain a consistent critical value.

In order to define the subsampling procedure, let 
\[ W_i = \{ X_{ki} : k = 1, 2, 3, 4, 5, 6 \} \]
for \( i = 1 \ldots N \). \( T_N \) denotes the test statistics \( D_N^{(s)} \). We first generate the subsamples of size \( b \) by taking without replacement from the original data. There will be \( N - b + 1 \) different subsamples of size \( b \). We then compute the test statistics \( t_{N,b,i} \) using subsamples \( \{ W_i, W_{i+1}, \ldots, W_{i+b-1} \} \) for \( i = 1, 2, \ldots, N - b + 1 \). Linton, Maasoumi, and Whang (2005) show that this subsampling procedure works under a very weak condition on \( b \) and is data-dependent. The sampling distribution \( G_N \) of \( T_N \) can be approximated by:

\[
G_{N,b}(w) = \frac{1}{N - b + 1} \sum_{i=1}^{N-b+1} 1(\sqrt{b} t_{N,b,i} \leq w). \tag{8}
\]

\( g_{N,b}(1 - \alpha) \) is the \( (1 - \alpha) \)th sample quantile of \( G_{N,b}(w) \). We reject the null at significant level \( \alpha \) if \( T_n > g_{N,b}(1 - \alpha) \).

5 Results

5.1 Summary Data on Mutual Funds

Table 2 reports the summary statistics for our actively managed mutual fund sample. There are a total of 2,666 mutual funds in our sample, which are divided into four categories as previously discussed. Aggressive funds attempt to achieve the highest capital gains and the investments held in these funds are companies that demonstrate high growth potential, usually accompanied by a large amount of share price volatility. Growth funds invest in growth companies with the primary aim of achieving capital gains instead of dividend in-
come. Income funds seek to provide a high current income by investing in high-yielding conservative stocks. Growth & income funds seek to provide both capital gains and a steady stream of income. In Panel A, we report the gross returns, net returns, skewness, kurtosis, autocorrelation, and Sharpe ratio for equal weighted mutual fund groups. Gross return is defined as the mutual funds’ return before deducting any management fees. Net return is the return received by investors. Consistent with what the previous literature has found, the average returns of all five mutual fund groups are lower than the market. The standard deviation for more conservative funds is lower. All mutual fund groups’ return and the market return are negatively skewed. All the returns series have some serial dependence based on the autocorrelation statistics. In Panel B, we report similar statistics for value-weighted mutual fund groups. Panel C shows that all of the returns of the mutual fund groups are highly correlated. Thus, the LMW stochastic dominance test is used because it accommodates not only general dependence between returns, but also serial dependence.

5.2 Normality Test

When the underlying variable is normal, the traditional performance evaluation measure will not miss anything by only considering mean and variance. However, one issue in performance evaluation is that the returns of mutual funds are non-normal. Table 3 shows the Kolmogorov-Smirnov and Jarque-Bera test results. For any group of mutual funds, the normality hypothesis is strongly rejected. Previous literature has also documented non-normalities in mutual fund returns. Kosowski, Timmermann, Wermers, and White (2006) suggest these non-normalities arise for three reasons. First, individual stocks within a typical mutual fund portfolio realize returns with non-negligible higher moments and managers often hold heavy positions in relatively few stocks or industries. Second, individual stocks exhibit varying levels of time-series autocorrelations in returns. Third, funds may implement dynamic strategies that involve changing their levels of risk-taking when the risk of the overall market portfolio changes. Kosowski, Timmermann, Wermers, and White (2006)
argue that normality may be a poor approximation in practice, even for a fairly large mutual fund portfolio. The stochastic dominance test is based on the entire distribution. Unlike the Sharpe ratio, it does not require the return to be normally distributed.

5.3 Mutual Funds and Market Return Comparison

Stochastic dominance tests implicitly take into account the differences in expected returns and risk. While traditional performance evaluation tools take the standard deviation as a quantifier for risk, the stochastic dominance approach will consider standard deviation, skewness, kurtosis, and all higher moments for the evaluation. For example, we are interested in comparing asset A and asset B for investors with general utility assumptions. If asset A has a higher expected return than asset B, then asset A will be preferred if we only consider the mean and ignore the risk. However, if the higher expected return of asset A is due to its higher risk, then asset A would exhibit more extreme positive and negative returns. For investors with various preferences for risk and return trade-off, asset A may or may not be preferred. Thus, asset A will not stochastically dominate asset B.

In this section, we apply the stochastic dominance test to compare the distributions of monthly market returns and mutual fund returns. Figure 1 shows the cumulative density function (CDF) of the realized equal weighted actively managed mutual fund returns and market returns from 1980-2015 and Figure 2 shows the CDF of the realized value-weighted actively managed mutual fund returns and market returns for the same time period. Inspection of the graph suggests no evidence of first order stochastic dominance as the two CDFs cross.

Table 4 summarizes the stochastic dominance test results for equal weighted mutual funds and the market. In Panel A, we test for stochastic dominance between the market and mutual fund net returns. In Panel B, we test for stochastic dominance between the market and mutual fund gross returns. The first column of Table 4 lists the return pairs we are testing. The null hypothesis is that the first return series will stochastically dominate the
second return series. For example, “Average Mutual Fund vs. Market” means that we test whether or not the equal-weighted average of mutual fund returns stochastically dominate the market. In the second column, we list the order of stochastic dominance being tested. The test statistics are given in the third column. The final three columns provide the p-value calculated from a different subsample block size.

The test statistics of FSD in Panel A of Table 4 has a value of 0.27 with a p-value of 0.00. As expected from Figure 1, the market returns do not dominate the average actively managed fund net returns by first order stochastic dominance. This implies that expected utility maximizers do not all prefer either actively managed mutual funds or the market benchmark. The test value of SSD in Panel A has a value of 0.01, with a p-value of 0.00, showing that there is also no evidence of second order stochastic dominance between the two assets. This implies that risk-averse investors do not all prefer either actively managed mutual funds or the market benchmark. The test value of TSD in Panel A is positive and shows no evidence of third order stochastic dominance between two assets. This implies that prudent investors also do not all prefer either actively managed mutual funds or the market benchmark.

Panel B of Table 4 shows the SD test results for the market and actively managed mutual fund gross returns. Even without deducting any management fees, there is still no evidence of a dominance relationship between two assets. The SD test statistics are all positive with p-values less than 5%.

Overall, the results in Table 4 show no stochastic dominance relationship between average actively managed mutual fund returns and the market returns by first order, second order, or third order stochastic dominance. The SD test statistics are all positive with p-values less than 5%. This suggests that investors with certain utility functions prefer the distribution of the market returns, while some other investors with different utility functions prefer the return distribution of actively managed mutual funds. The test results here reveal that investors’ utility functions will play a role in evaluating the return distribution of actively
managed funds and the market.

5.4 Investment Objective Subgroups of Mutual Funds and Market Return Comparison

Mutual funds have attempted to differentiate their services by specializing in certain sectors of the stock market and adopting various investment styles. For example, growth funds claim to specialize in low book-to-market stocks, while income funds claim to specialize in high book-to-market stocks. The question is whether such specialization adds value to investors. We investigate this issue by partitioning funds based on their self-declared investment objectives (aggressive, growth, income, and growth & income). In this Section, we use a stochastic dominance approach to examine whether some styles of mutual funds perform better than others or better than the market. Figure 2 and Figure 3 plot the CDF of four classes of mutual fund and the market returns. Once again, all of the CDFs cross, so we do not expect to find first order stochastic dominance.

Table 5 summarizes the stochastic dominance results for the four mutual fund classes and the market both before and after management fees have been deducted. Before deducting management fees, aggressive funds are third order stochastically dominated by each of the other three classes of mutual funds and also the market. After deducting management fees, aggressive funds are still third order dominated by each of the other three classes of mutual funds and second order dominated by the market. This test result shows that aggressive funds on average are inferior to the other three mutual fund classes and the market for all prudent investors with or without considering management fees. Also, on a net return basis, all risk-averse investors prefer the market to average aggressive funds. The underperformance of aggressive funds is not surprising given the high exposure to market risk and high betas. Hong and Sraer (2016) provide a theory for why high beta assets are prone to speculative overpricing. They point out that when investors disagree about the stock market’s prospects,
high beta assets are more sensitive to this aggregate disagreement. Thus, high beta assets experience a greater divergence of opinion about their payoffs and are overpriced due to short-sales constraints. The stochastic test result confirms that risk-averse individuals do not prefer aggressive funds. This suggests that the major flow to aggressive funds is probably made by investors with certain non-concave utility functions.

The absence of second order stochastic dominance between income funds and the market means that certain risk-averse individuals (e.g., those with quadratic utility functions) prefer the income fund, while some other risk-averse individuals with different utility functions prefer the market return. This result is in contrast to the Sharpe ratio result, presented in the summary statistics table, which posits that the income fund (Sharpe ratio 14.26) is preferred to the market (Sharpe ratio 13.98) for all agents with a quadratic utility function. Although the Sharpe ratio also considers this risk and return trade-off with variance as the quantifier for risk, since it ignores higher moments in the distribution, it does not provide an accurate result for all subsets of this data. In counterpoint, the stochastic dominance approach provides a robust analysis of the performance, which allows for differentiation between different types of investors.

Surprisingly, there is some evidence that both growth & income funds and income funds dominate the market by third order stochastic dominance before and also after fees. This implies that income and growth & income will be favored for all prudent individuals who have a preference for positive skewness and an aversion for variance and kurtosis. As shown in the summary statistics, income and growth & income funds have slightly lower average returns than the market. However, they both also have a lower variance, smaller negative skewness, and smaller kurtosis. Including these measures of risk preference will therefore provide a different picture of the fund performance evaluation. Even though these funds have lower returns, they are also less risky. The existence of third order stochastic dominance means that all prudent investors prefer income and growth & income funds to the market as seen in the entire 1980-2015 monthly return distribution.
5.5 Recession/Boom

The early literature on the value of active mutual fund management focuses on unconditional return performance and generally finds that the average fund underperforms passive benchmarks\(^8\) and that there is evidence of negative market timing.\(^9\) However, Moskowitz (2000), Kosowski (2006), and Glode (2011) all suggest that unconditional mutual fund performance measures may understate the value of mutual funds to investors since they cannot answer the question of how mutual funds perform in recession states when investors’ marginal utility of wealth is highest. Their findings imply that actively managed mutual funds perform better in recessions and are therefore potentially desirable relative to benchmarks. In this Section, we explore the performance of mutual funds and the market during different economics conditions. The stochastic dominance test is conducted for NBER recessions and expansions. Our aggregate sample spans 430 months of data from March 1980 until December 2015, among which 55 are NBER recession months (13%).

During economic expansion periods, the SD test results are very similar to what were seen in previous Sections. First, there is no dominance relationship between average actively managed mutual funds and the market by first order, second order, or third order stochastic dominance during the economic expansion periods in our sample. Second, Panel A of Table 6 shows that the market still dominates aggressive funds by second order stochastic dominance after deducting all the fees during economic expansion periods. Also, aggressive funds are third order stochastically dominated by the other three mutual fund classes. Third, there is evidence showing that income and growth & income funds dominate the market by third order dominance during economic expansions.


During economic recession periods, there is no dominance relationship between average actively managed mutual funds and the market by first order, second order, and third order stochastic dominance in our sample. Panel B of Table 6 shows the SD test results for the four styles of mutual funds and the market during economic recession periods. Aggressive funds are not only third order stochastically dominated by the market, but also second order stochastically dominated by income funds and the growth & income funds. This suggests that the underperformance of aggressive funds persist during recessions. Income and growth & income funds dominate the market by second order stochastic dominance. This implies that during recessions, risk-averse investors prefer growth & income funds and income funds to the market. Thus, these funds do create some value for risk-averse investors during economic recession periods.

5.6 Risk Adjusted Return

In order to further compare the performance among different classes of mutual funds, we calculate the risk adjusted return based on a four-factor model as proposed in Carhart (1997). The models use the regression framework below:

\[
R_{it} - R_{ft} = a_i + b_i(R_{Mt} - R_{ft}) + s_iSMB_t + h_iHML_t + m_iMOM_t + e_{it}.
\]

In this regression, \(R_{it}\) is the return on fund \(i\) for month \(t\), \(R_{ft}\) is the risk-free rate (the one month U.S. Treasury bill rate), \(R_{Mt}\) is the market return (the return on a VW portfolio of NYSE, Amex, and NASDAQ stocks), \(SMB_t\) and \(HML_t\) are the size and value factors as in Fama and French (1993), \(MOM_t\) is Carhart’s (1997) momentum factor, \(a_i\) is the average return left unexplained by the benchmark model, and \(e_{it}\) is the regression residual. Table 7 provides the summary statistics for all of the factors used in the regression and Table 8 shows the regression results. Overall, mutual funds do tilt their investments more toward stocks that match their stated objectives. Aggressive funds have more exposure to all risk factors. It is well-known that aggressive funds tilt toward small capitalization, low book-to-market, and momentum stocks, while the opposite holds true for income funds.
For each fund $i$, the risk-adjusted return is calculated as:

$$\hat{\alpha}_{it} = R_{it} - \hat{\beta}^T_i Z_t,$$

where $Z_t$ is the value of factors at month $t$.

We next conduct an analysis of the distributions of risk adjusted returns of the mutual funds. Table 9 shows the SD test results for risk adjusted returns based on the four-factor model. After controlling the market risk premium, size, value, and momentum factors, the risk adjusted returns of aggressive funds are dominated by all of the other three classes of mutual funds by second order stochastic dominance. In addition, the risk adjusted returns of growth & income funds dominate all of the other three classes of mutual funds by second order stochastic dominance.

### 5.7 Investment Strategy

Two important issues for mutual fund investors are whether a superior mutual fund can be identified in advance and whether the superior performance persists. Many studies have found performance persistence in the top-ranked mutual fund groups based on past returns, past alpha, and past Sharpe ratio.$^{10}$ In this Section, we use the stochastic dominance relationship as a criterion for portfolio construction. We examine whether ex-post SD relationships provide exploitable information on ex-ante returns. This empirical exercise targets second order stochastic dominance. At the beginning of each year between 1995-2015, we identify the undominated (second order) mutual funds based on the most recent 60-month returns. We then form an equal weighted portfolio of undominated mutual funds. The portfolio is rebalanced annually. For comparison, mean-variance efficient portfolios are formed for the same time period.

Table 10 shows the portfolio performance based on a stochastic dominance approach and

a mean-variance approach. The mean return of the portfolio of second order undominated funds is 1.92%, which is substantially larger than the portfolio of first order dominated funds. The average return of the mean-variance efficient portfolio is 1.42%, with a 3.21 standard deviation and negative skewness. The portfolio of second order undominated funds has a smaller standard deviation and positive skewness compared to the mean-variance efficient portfolio. This shows that the stochastic dominance approach may potentially be used for mutual fund selection.

6 Robustness

6.1 Liquidity Factor

Pástor and Stambaugh (2003) show that expected stock returns are related cross-sectionally to the sensitivities of the returns to fluctuations in aggregate liquidity. We introduce the liquidity factor to capture such an effect, in addition to the market, size, value, and momentum factors. Table 11 shows the SD test results for risk adjusted returns based on a five-factor model. The result is similar to what we have before. After controlling for the market risk premium, size, value, momentum, and liquidity factors, the risk adjusted returns of aggressive funds are dominated by all of the other three classes of mutual funds by second order stochastic dominance. Also, the risk adjusted returns of growth & income funds dominate all of the other three classes of mutual funds by second order stochastic dominance.

6.2 Value Weighted Portfolios

As a robustness check, we consider if our results are sensitive to the weighting method. We perform all of the analyses again using the value-weighted mutual fund portfolios. Figure 4 plots the CDF of the net and gross return distributions of the market and the value-weighted mutual fund portfolios. As before, the two CDF’s cross and we do not expect to find a first
order stochastic dominance relationship. Overall, we found the results are very robust to different weighting methods. First, Table 12 shows that there is no stochastic dominance relationship between value-weighted mutual fund portfolios and the market, with or without fees.

Second, the results in Table 13 show that the market still dominates aggressive funds by second order dominance after deducting all fees. Also, aggressive funds are third order stochastically dominated by all of the other three mutual fund classes. Third, there is evidence showing that income and growth & income funds dominate the market by third order dominance, with or without deducting the management fees.

Finally, Table 14 shows the SD test results for value-weighted risk adjusted returns based on four-factor and five factor models. In both cases, the risk adjusted returns of aggressive funds are dominated by all of the other three classes of mutual funds by third order stochastic dominance. In addition, the risk adjusted returns of growth & income funds dominate both growth funds and income funds by second order stochastic dominance.

7 Conclusion

Although there is no consensus on investors’ utility function form, traditional mutual fund performance evaluation measures usually rely on a quadratic utility assumption. Moreover, even though investors recognize the importance of the higher moments of a return distribution, they generally only use variance as a risk measurement. To address this issue, this paper evaluates mutual fund performance using a non-parametric framework that 1) imposes a minimal set of conditions on preferences; and 2) analyzes the entire return distribution for each mutual fund group. Previous literature finds that actively managed mutual funds on average underperform the passive benchmark by comparing the mean and standard deviation of returns. We revisit the actively managed mutual funds underperformance puzzle by applying the stochastic dominance test proposed by Linton, Maasoumi, and Whang (2005)
to verify if actively managed mutual funds on average underperform and if any particular style of actively managed mutual funds (aggressive, growth, growth & income, and income) underperforms. The test results show little evidence that actively managed mutual funds on average underperform the passive benchmark. This suggests that investors with different utility functions will have different preferences over actively managed mutual funds and the passive benchmark. Although aggressive mutual funds underperform the market for risk-averse investors, there is some evidence showing that both growth & income and income funds outperform the market for prudent investors. Furthermore, we find that mutual fund portfolios formed by the stochastic dominance approach provide superior future performance.

Reference


The Case of Hedge Funds”, Review of Financial Studies, 10 (1997), 275-302


Table 1: Mutual Fund Style Classification

The CRSP U.S. Survivor-Bias-Free Mutual Funds database includes style and objective codes from three different sources over the life of the database. No single source exists for its full-time range. Wiesenberger Objective codes are populated between 1962-1993; Strategic Insight Objective codes are populated between 1993-1998; and Lipper Objective codes begin in 1998. We classify mutual funds with the objective of “Maximum Capital Gains,” “Equity USA Aggressive Growth,” “Capital Appreciation Funds” as aggressive funds. Mutual funds with the objective of “Growth,” “Long-Term Growth,” and “Equity USA Growth” are growth funds. Mutual funds with the objective of “Equity Income,” “Option Income,” and “Equity Income Funds” are income funds. Mutual funds with the objective of “Growth and Current Income,” “Equity USA growth & income,” “Equity USA Income & Growth,” and “Growth and Income Funds” are growth & income funds.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>MCG Maximum Capital Gains</td>
<td>AGG Equity USA Aggressive Growth</td>
<td>CA Capital Appreciation Funds</td>
</tr>
<tr>
<td>Growth</td>
<td>G Growth; LTG Long-Term Growth</td>
<td>GRO Equity USA Growth</td>
<td>G Growth Funds</td>
</tr>
<tr>
<td>Income</td>
<td>IEQ Equity Income</td>
<td>OPI Option Income</td>
<td>EI Equity Income Funds</td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>GCI Growth and Current Income</td>
<td>GRI Equity USA Growth &amp; Income; ING Equity USA Income &amp; Growth</td>
<td>GI Growth and Income Funds</td>
</tr>
</tbody>
</table>
Table 2: Summary statistics
This table reports the summary statistics for the funds in our sample. The sample period is March 1980-December 2015. Mutual fund share class level returns are from the CRSP mutual fund database. We combined different classes of the same fund into a single fund using the identification in MFLINKS. Each monthly fund return is computed by weighting the return of its component share classes by their beginning-of-month total net asset values. “Number of funds” is the number of mutual funds that meet our selection criteria for being an active mutual fund and have a self-declared investment objective of “MCG,” “AGG,” “CA,” “G,” “LTG,” “GRO,” “IEQ,” “GPI,” “EI,” “GCI,” “GRI,” or “GI.” Gross return is the mutual fund’s return before deducting any management fees. Net return is the return received by investors. Market return (column 7) reports the returns on a VW portfolio of NYSE, Amex, and NASDAQ stocks.

<table>
<thead>
<tr>
<th>Panel A: EW</th>
<th>Aggressive</th>
<th>Growth</th>
<th>G&amp;I</th>
<th>Income</th>
<th>All</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Return (%/month)</td>
<td>1.05</td>
<td>1.00</td>
<td>0.98</td>
<td>0.96</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Net Return (%/month)</td>
<td>0.93</td>
<td>0.91</td>
<td>0.90</td>
<td>0.88</td>
<td>0.91</td>
<td>1.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.93</td>
<td>4.40</td>
<td>3.93</td>
<td>3.58</td>
<td>4.30</td>
<td>4.48</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.37</td>
<td>5.76</td>
<td>5.29</td>
<td>5.31</td>
<td>5.60</td>
<td>5.33</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.71</td>
<td>-0.83</td>
<td>-0.68</td>
<td>-0.71</td>
<td>-0.81</td>
<td>-0.73</td>
</tr>
<tr>
<td>Number of Funds</td>
<td>347</td>
<td>1573</td>
<td>635</td>
<td>111</td>
<td>2666</td>
<td></td>
</tr>
<tr>
<td>Minimum (%/month)</td>
<td>-25.08</td>
<td>-23.13</td>
<td>-19.18</td>
<td>-16.78</td>
<td>-22.65</td>
<td>-22.64</td>
</tr>
<tr>
<td>Maximum (%/month)</td>
<td>13.69</td>
<td>11.72</td>
<td>10.65</td>
<td>10.33</td>
<td>11.83</td>
<td>12.89</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.13</td>
<td>0.10</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
<td>0.08</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Panel B: VW</th>
<th>Aggressive</th>
<th>Growth</th>
<th>G&amp;I</th>
<th>Income</th>
<th>All</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Return (%/month)</td>
<td>1.07</td>
<td>1.02</td>
<td>0.98</td>
<td>0.98</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Net Return (%/month)</td>
<td>0.98</td>
<td>0.94</td>
<td>0.93</td>
<td>0.91</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.92</td>
<td>4.51</td>
<td>3.83</td>
<td>3.79</td>
<td>4.25</td>
<td>4.48</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.45</td>
<td>5.44</td>
<td>5.29</td>
<td>5.24</td>
<td>5.57</td>
<td>5.33</td>
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<tr>
<td>Skewness</td>
<td>-0.70</td>
<td>-0.75</td>
<td>-0.71</td>
<td>-0.73</td>
<td>-0.76</td>
<td>-0.73</td>
</tr>
<tr>
<td>Maximum (%/month)</td>
<td>15.03</td>
<td>12.42</td>
<td>11.18</td>
<td>10.43</td>
<td>12.04</td>
<td>12.89</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.12</td>
<td>0.09</td>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
<td>0.08</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Correlation</th>
<th>Aggressive</th>
<th>Growth</th>
<th>Growth &amp; Income</th>
<th>Income</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>0.98</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>0.93</td>
<td>0.98</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>0.87</td>
<td>0.93</td>
<td>0.98</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
<td>0.95</td>
<td>1.00</td>
</tr>
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</table>
Table 3: Normality Test for Mutual Fund Returns

This table shows the normality test results for mutual fund returns. The sample period is March 1980-December 2015. Mutual fund share class level returns are from the CRSP mutual fund database. We combined different classes of the same fund into a single fund using the identification in MFLINKS, with value weights. The null hypothesis is $H_0$: Data follows a normal distribution. The alternative hypothesis is that $H_a$: Data does not follow a normal distribution. The test results show that the normality assumption is strongly rejected by the test.

<table>
<thead>
<tr>
<th>Test</th>
<th>Kolmogorov-Smirnov Test statistics</th>
<th>P value</th>
<th>Jarque-Bera Pr(skew)</th>
<th>Jarque-Bera Pr(Kurt)</th>
<th>P value</th>
</tr>
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<tbody>
<tr>
<td>Aggressive</td>
<td>0.43</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Growth</td>
<td>0.44</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>0.44</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Income</td>
<td>0.42</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>All</td>
<td>0.43</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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</tbody>
</table>
Figure 1: CDF of EW Mutual Funds and Market Returns
This figure plots the CDF of EW mutual fund and market returns. In the first Panel, the solid blue line is the CDF of the market returns and the red line is the CDF of EW mutual fund net returns. In the second Panel, the solid blue line is the CDF of the market returns and the red line is the CDF of EW mutual fund gross returns. The sample period is from March 1980 and December 2015.
Table 4: Stochastic Dominance Test Statistics for EW Mutual Fund and Market Returns

This Table shows the stochastic dominance test results between the market and equally-weighted mutual fund returns. The sample includes all domestic actively managed equity mutual funds in the CRPS-MFLINK merged dataset from March 1980-December 2015. Panel A reports the SD test results for net returns and Panel B reports the test results for gross returns. The P-value is based on subsampling, which takes samples without replacement of various block sizes from the original sample. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

<table>
<thead>
<tr>
<th>Test Stat</th>
<th>Subsample Block Size</th>
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<tr>
<td><strong>Panel A: Market and EW Mutual Fund Net Returns</strong></td>
<td></td>
</tr>
<tr>
<td>Average Mutual Fund vs. Market</td>
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<tr>
<td>FSD</td>
<td>0.58</td>
</tr>
<tr>
<td>SSD</td>
<td>0.01</td>
</tr>
<tr>
<td>TSD</td>
<td>0.001</td>
</tr>
<tr>
<td>Market vs. Average Mutual Fund</td>
<td></td>
</tr>
<tr>
<td>FSD</td>
<td>0.27</td>
</tr>
<tr>
<td>SSD</td>
<td>0.01</td>
</tr>
<tr>
<td>TSD</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>Panel B: Market and EW Mutual Funds Gross Returns</strong></td>
<td></td>
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<tr>
<td>Average Mutual Fund vs. Market</td>
<td></td>
</tr>
<tr>
<td>FSD</td>
<td>0.41</td>
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<tr>
<td>SSD</td>
<td>0.01</td>
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<tr>
<td>TSD</td>
<td>0.001</td>
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<tr>
<td>Market vs. Average Mutual Fund</td>
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<tr>
<td>FSD</td>
<td>0.38</td>
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<tr>
<td>SSD</td>
<td>0.01</td>
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<tr>
<td>TSD</td>
<td>0.001</td>
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</table>
Figure 2: CDF of Aggressive and Growth Funds Returns
This Figure plots the CDF of aggressive and growth fund returns. In the first Panel, the solid blue line is the CDF of the market returns and the red line is the CDF of aggressive fund returns. In the second Panel, the solid blue line is the CDF of the market returns and the red line is the CDF of growth fund returns. The sample period is from March 1980 and December 2015.
Figure 3: CDF of Growth & Income and Income Funds Returns
This Figure plots the CDF of growth & income and income fund returns. In the first Panel, the solid blue line is the CDF of the market returns and the red line is the CDF of growth & income fund returns. In the second Panel, the solid blue line is the CDF of the market returns and the red line is the CDF of income fund returns. The sample period is from March 1980 and December 2015.
Table 5: Stochastic Dominance Test Results for the Market and Four Mutual Fund Classes

This Table reports the stochastic dominance test results for returns of four mutual fund classes and the market. An entry in the table means that the mutual fund style on the left dominates the mutual fund style/market at the top. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

Panel A: EW Net Returns

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Market</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>TSD</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Market</td>
<td>SSD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: EW Gross Returns

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Market</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>TSD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>TSD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
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</tr>
<tr>
<td>Growth &amp; Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
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</table>
Table 6: Fund Returns During Recessions and Expansions

This Table reports the stochastic dominance test results for the returns of four mutual fund classes and the market during NBER recessions and NBER expansions. Our aggregate sample spans 430 months of data from March 1980 until December 2015, among which 55 are NBER recession months (13%). An entry in the table means that the mutual fund style on the left dominates the mutual fund style/market at the top. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

<table>
<thead>
<tr>
<th>Panel A: EW Net Returns during NBER Expansions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Aggressive</td>
</tr>
<tr>
<td>Growth</td>
</tr>
<tr>
<td>Market</td>
</tr>
<tr>
<td>Income</td>
</tr>
<tr>
<td>Growth &amp; Income</td>
</tr>
<tr>
<td>Aggressive</td>
</tr>
<tr>
<td>TSD</td>
</tr>
<tr>
<td>SSD</td>
</tr>
<tr>
<td>TSD</td>
</tr>
<tr>
<td>TSD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: EW Net Returns during NBER Recessions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Aggressive</td>
</tr>
<tr>
<td>Growth</td>
</tr>
<tr>
<td>Market</td>
</tr>
<tr>
<td>Income</td>
</tr>
<tr>
<td>Growth &amp; Income</td>
</tr>
<tr>
<td>Aggressive</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>TSD</td>
</tr>
<tr>
<td>SSD</td>
</tr>
<tr>
<td>SSD</td>
</tr>
</tbody>
</table>
Table 7: Summary Statistics for Monthly Explanatory Returns for Four-factor and Five-factor Models

$R_M$ is the return on a value-weighted market portfolio of NYSE, Amex, and NASDAQ stocks and $R_f$ is the 1-month Treasury bill rate. The construction of $SMB_t$ and $HML_t$ follows Fama and French (1993). The momentum return, $MOM_t$, is the simple average of the month $t$ returns on the two high momentum portfolios minus the average of the returns on the two low momentum portfolios. The construction of $Liquidity_t$ follows Pástor and Stambaugh (2003). All of the factors are obtained through WRDS. The Table shows the average monthly returns, the standard deviation of monthly returns, and the t-statistic for the average monthly returns. The period is March 1980 through December 2015.

<table>
<thead>
<tr>
<th></th>
<th>$R_M - R_f$</th>
<th>$SMB_t$</th>
<th>$HML_t$</th>
<th>$MOM_t$</th>
<th>$Liquidity_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.63</td>
<td>0.12</td>
<td>0.26</td>
<td>0.61</td>
<td>0.51</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(4.49)</td>
<td>(3.05)</td>
<td>(3.01)</td>
<td>(4.56)</td>
<td>(3.67)</td>
</tr>
</tbody>
</table>
Table 8: Performance of Equally-weighted Portfolio of Funds

This Table provides the four-factor model regression result for the entire actively managed equity mutual fund population, as well as for aggressive, growth, growth and income, and income funds. The regression are based on monthly data between March 1980 and December 2015. Each Panel contains the estimated alpha, the estimated exposures to the market, size, value, and momentum factors. Figures below are the coefficient value denote the Newey–West (1987) heteroskedasticity and autocorrelation consistent estimates of p-values under the null hypothesis that the regression parameters are equal to zero.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\alpha}$ (annual)</th>
<th>$\hat{\beta}_m$</th>
<th>$\hat{\beta}_{smb}$</th>
<th>$\hat{\beta}_{hml}$</th>
<th>$\hat{\beta}_{mon}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>-0.82%</td>
<td>0.98</td>
<td>0.31</td>
<td>-0.09</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Growth</td>
<td>-0.73%</td>
<td>0.96</td>
<td>0.10</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.15)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>-0.59%</td>
<td>0.91</td>
<td>-0.05</td>
<td>0.14</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Income</td>
<td>-0.74%</td>
<td>0.95</td>
<td>-0.09</td>
<td>0.26</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>All</td>
<td>-0.72%</td>
<td>0.94</td>
<td>0.07</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>
Table 9: Four-factor Risk Adjusted Return Performance

This Table reports the stochastic dominance test result for four-factor model risk adjusted returns for the four classes of mutual funds. An entry in the Table means that the mutual fund style on the left dominates the mutual fund style at the top. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance. The sample period is from March 1980 through December 2015.

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>SSD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>SSD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>SSD</td>
<td>SSD</td>
<td>SSD</td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Equal Weighted Risk Adjusted Net Returns Based on Four Factor Models
Table 10: Investment Strategy Based on SD and MV approach
This Table reports the summary statistics of returns for portfolios of second order undominated funds, first order dominated funds, and MV efficient mutual funds between 1995-2015. At the beginning of each year, we form mutual fund portfolios based on the stochastic dominance or the mean-variance efficient test results of the most recent 60 month returns. We hold the portfolio for one year and rebalance annually. $S_U$ denotes second order undominated funds, $F_D$ denotes first order dominated funds. Column 2 shows the average number of mutual funds held in each portfolios. Column 3-6 reports the summary statistics of a portfolio’s monthly equal-weighted net return.

<table>
<thead>
<tr>
<th>No. of funds</th>
<th>Net return (%/Month)</th>
<th>Std.Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_U$</td>
<td>45</td>
<td>1.92%</td>
<td>3.02</td>
<td>0.12</td>
</tr>
<tr>
<td>$F_D$</td>
<td>1016</td>
<td>0.11%</td>
<td>4.61</td>
<td>-0.97</td>
</tr>
<tr>
<td>MV efficient</td>
<td>67</td>
<td>1.42%</td>
<td>3.21</td>
<td>-0.54</td>
</tr>
</tbody>
</table>
Table 11: Five-factor Risk Adjusted Return
This Table reports the stochastic dominance test result for five-factor model risk adjusted returns for four classes of mutual funds. An entry in the Table means that the mutual fund style on the left dominates the mutual fund style at the top. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

Panel A: Equal Weighted Risk Adjusted Net Return Based on Five Factor Model

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>SSD</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>SSD</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>SSD</td>
<td>SSD</td>
<td>SSD</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 4: CDF for VW Mutual Fund and Market Returns
This Figure plots the CDF of the VW mutual fund and market returns. In the first Panel, the solid blue line is the CDF of market returns and the red line is the CDF of VW mutual fund net returns. In the second Panel, the solid blue line is the CDF of market returns and the red line is the CDF of VW mutual fund gross returns. The sample period is from March 1980 and December 2015.
Table 12: Stochastic Dominance Test Statistics for VW Mutual Fund and Market Returns

This Table shows the stochastic dominance test results between the market and value-weighted mutual fund returns. The sample includes all domestic actively managed equity mutual funds in the CRPS-MFLINK merged dataset from March 1980-December 2015. Panel A reports the stochastic dominance test results for net returns and Panel B reports the stochastic dominance test results for the gross return. P-values are based on subsampling, which takes samples without replacement of various block sizes from the original sample. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

<table>
<thead>
<tr>
<th>Panel A: Market v.s. VW Mutual Fund Net Returns</th>
<th>Test Stat</th>
<th>Subsample Block Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Average Mutual Fund</td>
<td>FSD</td>
<td>0.44</td>
</tr>
<tr>
<td>vs. Market</td>
<td>SSD</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>TSD</td>
<td>0.001</td>
</tr>
<tr>
<td>Market</td>
<td>FSD</td>
<td>0.31</td>
</tr>
<tr>
<td>vs. Average Mutual Fund</td>
<td>SSD</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>TSD</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel B: Market v.s. VW Mutual Fund Gross Returns</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Average Mutual Fund</td>
<td>FSD</td>
<td>0.37</td>
</tr>
<tr>
<td>vs. Market</td>
<td>SSD</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>TSD</td>
<td>0.001</td>
</tr>
<tr>
<td>Market</td>
<td>FSD</td>
<td>0.34</td>
</tr>
<tr>
<td>vs. Average Mutual Fund</td>
<td>SSD</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>TSD</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Table 13: Stochastic Dominance Test Result for the Market and Four Mutual Fund Classes

This Table reports the stochastic dominance test results for the returns of the four mutual fund classes and the market. An entry in the table means that the mutual fund style on the left dominates the mutual fund style/market at the top. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Market</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: VW Net Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>TSD</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>SSD</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Market</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: VW Gross Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>TSD</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>TSD</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>TSD</td>
<td>TSD</td>
<td>TSD</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
Table 14: Risk Adjusted Return Performance for Value-weighted Mutual Funds

This Table reports the stochastic dominance test results for four-factor model and five-factor model risk adjusted returns for four classes of mutual funds. An entry in the table means that the mutual fund style on the left dominates the mutual fund style at the top. FSD denotes first order, SSD denotes second order, and TSD denotes third order stochastic dominance.

Panel A: Value Weighted Risk Adjusted Net Return Based on Four-factor Model

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>TSD</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>TSD</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>TSD</td>
<td>SSD</td>
<td>SSD</td>
<td>-</td>
</tr>
</tbody>
</table>

Panel B: Value Weighted Risk Adjusted Net Return Based on Five-factor Models

<table>
<thead>
<tr>
<th></th>
<th>Aggressive</th>
<th>Growth</th>
<th>Income</th>
<th>Growth &amp; Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>TSD</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>TSD</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth &amp; Income</td>
<td>TSD</td>
<td>SSD</td>
<td>SSD</td>
<td>-</td>
</tr>
</tbody>
</table>